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Soft union-plus product of groups

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ABSTRACT A rigorous and expressive algebraic framework for modeling systems with uncertainty, ambiguity, and parameter-dependent variability is provided by soft set theory. In this paper, we present a new binary operation on soft sets whose parameter domains have group-theoretic structure: the soft union–plus product. The operation is completely compatible with generalized concepts of soft equality and soft subsethood when specified formally inside an axiomatic framework. Key structural aspects such as closure, associativity, commutativity, idempotency, and distributivity are investigated in detail algebraically, includinng its behavior with respect to identity, absorbing, null, and absolute soft sets. The outcomes demonstrate that the operation creates a strong and cohesive algebraic system on the universe of soft sets while adhering to all algebraic limitations imposed by group-indexed domains. In addition to its theoretical importance, the operation provides a strong basis for a generalized soft group theory and reinforces the underlying algebraic architecture of soft set theory. Furthermore, it has significant potential for both abstract theoretical advancement and real-world applications due to its formal consistency with soft subset and equality relations, which improves its usefulness in domains like categorization, decision-making, and uncertainty-aware modeling.

Keywords: Soft sets, Soft subsets, Soft equalities, Soft union-plus product.

1. INTRODUCTION

In order to deal with ambiguity, indeterminacy, and uncertainty in a variety of fields, including engineering, economics, the social sciences, and medical diagnostics, numerous mathematically precise frameworks have been devised. However, fundamental models such as probabilistic systems and fuzzy set theory [1] have inherent limitations: probabilistic models assume precise distributions and reproducible conditions, which are often broken in real-world applications, while fuzzy sets rely on subjective membership functions.

Molodtsov [2] responded by putting out soft set theory, a framework based on parameters that circumvents these limitations. Basic operations were introduced by Maji et al. [3], then recast from an information-theoretic perspective by Pei and Miao [4], greatly advancing the theory. Following contributions [6–19] that resolved ambiguities, enlarged the algebra of soft operations, and generalized soft equalities, Ali et al. [5] enhanced the theory's operational flexibility through restricted and extended operations. This fundamental framework has been significantly enhanced in recent years by the methodical introduction and formal algebraic analysis of recently described operations. The contributions of [20–34] are especially noteworthy, as their combined efforts have produced a strong, extensible, and internally consistent algebraic framework that sustains and advances the theoretical development of soft set theory.

Parallel developments centered on soft subsethood and equality, which were initially generalized by Pei and Miao [4], Feng et al. [35], and Qin and Hong [36] with further extensions by Jun and Yang [37] and Liu et al. [38] through J-soft and L-soft equalities. Feng and Yongming [39] classified soft subsets under L-equality, showing that certain quotient structures yield semigroup properties. Broader generalizations such as g-soft, gf-soft, and T-soft equalities were introduced in [40-43] embedding congruence and lattice-theoretic mechanisms into soft algebra. A pivotal reformulation was made by Çağman and Enginoğlu [44], who established a consistent axiomatic basis for soft sets. Building on this, binary operations have been defined over classical algebraic structures: soft intersection-union products have been extended to rings [45], semigroups [46], and groups [47], while the dual soft union-intersection product has been explored in group-theoretic [48], semigroup-theoretic [49], and ring-theoretic [50] settings.

Building on this rich landscape, the present study introduces the soft union-plus product, a novel binary operation defined over soft sets indexed by group-structured parameter domains. Constructed within a formally consistent axiomatic system, the operation satisfies closure, associativity, commutativity, idempotency, and distributivity. Its behavior with respect to identity, absorbing, null, and absolute soft sets is rigorously analyzed. Importantly, it aligns with generalized soft subsethood and equality, enabling integration into existing soft algebraic frameworks. A comparative evaluation with earlier soft operations highlights its expressive strength and structural consistency across soft subset hierarchies. This contribution extends the algebraic foundations of soft set theory by emulating classical group-theoretic behavior within a soft context, thus offering a robust basis for developing generalized soft group theory. It further supports applications in abstract algebra, algebraic classification, and uncertainty-aware computation, reinforcing both the theoretical depth and practical utility of soft set frameworks.

2. METHOD

This section rigorously re-establishes the foundational definitions and algebraic postulates that underpin the theoretical framework developed herein. Initially introduced by Molodtsov [2] as a parameter-dependent formalism for modeling epistemic uncertainty, soft set theory lacked the algebraic rigor required for formal advancement. This shortcoming was effectively addressed in [44], whose axiomatic refinement resolved structural inconsistencies and established a coherent and algebraically robust foundation. The present study is built entirely upon this revised framework, which serves as the basis for all ensuing definitions, operations, and algebraic constructions. Unless stated otherwise, all subsequent references to soft sets and their associated operations are to be understood within the context of this refined formalism.

Definition 2.1. [44] Let E be a parameter set, U be a universal set, $P(U)$ be the power set of U, and $\mathcal{H} \subseteq E$. Then, the soft set $f_{\mathcal{H}}$ over U is a function such that $f_{\mathcal{H}}: E \to P(U)$, where for all $w \notin \mathcal{H}$, $f_{\mathcal{H}}(w) = \emptyset$. That is,

$$f_{\mathcal{H}} = \{(w, f_{\mathcal{H}}(w)) : w \in E\}$$

From now on, the soft set over U is abbreviated by $\mathcal{SS}$.

Definition 2.2. [44] Let $f_{\mathcal{H}}$ be an $\mathcal{SS}$. If $f_{\mathcal{H}}(w) = \emptyset$ for all $w \in E$, then $f_{\mathcal{H}}$ is called a null SS and indicated by $\emptyset_E$, and if $f_{\mathcal{H}}(w) = U$, for all $w \in E$, then $f_{\mathcal{H}}$ is called an absolute SS and indicated by U_E.

Definition 2.3. [44] Let $f_{\mathcal{H}}$ and $g_{\aleph}$ be two $\mathcal{SS}$s. If $f_{\mathcal{H}}(w) \subseteq g_{\aleph}(w)$, for all $w \in E$, then $f_{\mathcal{H}}$ is a soft subset of $g_{\aleph}$ and indicated by $f_{\mathcal{H}} \tilde{\subseteq} g_{\aleph}$. If $f_{\mathcal{H}}(w) = g_{\aleph}(w)$, for all $w \in E$, then $f_{\mathcal{H}}$ is called soft equal to $g_{\aleph}$, and denoted by $f_{\mathcal{H}} = g_{\aleph}$.

Definition 2.4. ([44] Let $f_{\mathcal{H}}$ and $g_{\aleph}$ be two $\mathcal{SS}$s. Then, the union of $f_{\mathcal{H}}$ and $g_{\aleph}$ is the $\mathcal{SS}$ $f_{\mathcal{H}} \tilde{\cup} g_{\aleph}$, where $(f_{\mathcal{H}} \tilde{\cup} g_{\aleph})(w) = f_{\mathcal{H}}(w) \cup g_{\aleph}(w)$, for all $w \in E$.

Definition 2.5. [44] Let $f_{\mathcal{H}}$ be an $\mathcal{SS}$. Then, the complement of $f_{\mathcal{H}}$, denoted by ${f_{\mathcal{H}}}^{c}$, is defined by the soft set ${f_{\mathcal{H}}}^{c}: E \to P(U)$ such that ${f_{\mathcal{H}}}^{c}(e) = U \setminus f_{\mathcal{H}}(e) = (f_{\mathcal{H}}(e))'$, for all $e \in E$.

Definition 2.6. [51] Let f_K and $g_{\aleph}$ be two $\mathcal{SS}$s. Then, f_K is called a soft S-subset of $g_{\aleph}$, denoted by $f_K \tilde{\subseteq}_S g_{\aleph}$ if for all $w \in E$, $f_K(w) = \mathcal{M}$ and $g_{\aleph}(w) = \mathcal{D}$, where $\mathcal{M}$ and $\mathcal{D}$ are two fixed sets and $\mathcal{M} \subseteq \mathcal{D}$. Moreover, two SSs f_K and $g_{\aleph}$ are said to be soft S-equal, denoted by $f_K =_S g_{\aleph}$, if $f_K \tilde{\subseteq}_S g_{\aleph}$ and $g_{\aleph} \tilde{\subseteq}_S f_K$.

It is obvious that if $f_K =_S g_{\aleph}$, then f_K and $g_{\aleph}$ are the same constant functions, that is, for all $w \in E$, $f_K(w) = g_{\aleph}(w) = \mathcal{M}$, where $\mathcal{M}$ is a fixed set .

Definition 2.7. [51] Let f_K and $g_{\aleph}$ be two $\mathcal{SS}$s. Then, f_K is called a soft A-subset of $g_{\aleph}$, denoted by $f_K \tilde{\subseteq}_A g_{\aleph}$, if, for each $a, b \in E$, $f_K(a) \subseteq g_{\aleph}(b)$.

Definition 2.8. [51] Let f_K and $g_{\aleph}$ be two SSs. Then, f_K is called a soft S-complement of $g_{\aleph}$, denoted by $f_K =_S (g_{\aleph})^c$, if, for all $w \in E$, $f_K(w) = \mathcal{M}$ and $g_{\aleph}(w) = \mathcal{D}$, where $\mathcal{M}$ and $\mathcal{D}$ are two fixed sets and $\mathcal{M} = \mathcal{D}'$. Here, $\mathcal{D}' = U \setminus \mathcal{D}$.

From now on, let G be a group, and $S_G(U)$ denotes the collection of all SSs over U, whose parameter sets are G; that is, each element of $S_G(U)$ is an SS parameterized by G.

Definition 2.9. [52] Let f_G and g_G be two SSs. Then, the soft intersection-difference product $f_G \otimes_{i/d} g_G$ is defined by

$$\left(f_G \otimes_{i/d}\right)(x) = \bigcap_{x=yz} \left(f_G(y) \setminus g_G(z)\right), \qquad y, z \in G$$

for all $x \in G$.

For additional information on SSs, we refer to [53-78].

3. RESULTS AND DISCUSSION

This section introduces and formally defines a novel binary operation on soft sets, termed the soft union-plus product, constructed over parameter domains equipped with a group-theoretic structure. A thorough algebraic investigation is conducted to establish its core structural properties—closure, associativity, commutativity, and idempotency—as well as its compatibility with generalized notions of soft equality and subsethood. Particular emphasis is placed on the operation's behavior within soft inclusion hierarchies and its coherence with the broader algebraic landscape of soft set theory.

Definition 3.1. Let f_G and g_G be two SSs over U. Then, the soft union-plus product $f_G \otimes_{u/p} g_G$ is defined by

$$\left(f_G \otimes_{u/p} g_G\right)(x) = \bigcup_{x=yz} \left(f_G(y) + g_G(z)\right) = \bigcup_{x=yz} \left(\left(f_G(y)\right)' \cup g_G(z)\right), \qquad y, z \in G$$

for all $x \in G$.

Note here that since G is a group, there always exist $y, z \in G$ such that $x = yz$, for all $x \in G$. Let the order of the group G be n, that is, $|G| = n$. Then, it is obvious that there exist n distinct representations for each $x \in G$ such that $x = yz$, where $y, z \in G$. Besides, for more on plus $(+)$ operation of sets, we refer to [53].

Note 3.2. The soft union-plus product is well-defined in $S_G(U)$. In fact, let f_G, g_G, σ_G, $k_G \in S_G(U)$ such that $(f_G, g_G) = (\sigma_G, k_G)$. Then, $f_G = \sigma_G$ and $g_G = k_G$, implying that $f_G(x) = \sigma_G(x)$ and $g_G(x) = k_G(x)$ for all $x \in G$. Thereby, for all $x \in G$.

$$\begin{aligned}\left(f_G \otimes_{u/p} g_G\right)(x) &= \bigcup_{x=yz} \left({f_G}^c(y) \cup g_G(z)\right) \\ &= \bigcup_{x=yz} \left({\sigma_G}^c(y) \cup k_G(z)\right) \\ &= \left(\sigma_G \otimes_{u/p} k_G\right)(x)\end{aligned}$$

Hence, $f_G \otimes_{u/p} g_G = \sigma_G \otimes_{u/p} k_G$.

Example 3.3. Consider the group $G = \{Ձ, Ϭ\}$ with the following operation**:**

·	Ձ	Ϭ
Ձ	Ձ	Ϭ
Ϭ	Ϭ	Ձ

Let f_G and g_G be two SSs over $U = D_2 = \{< x, y >: x^2 = y^2 = e, xy = yx\} = \{e, x, y, yx\}$ as follows:

$f_G = \{(Ձ, \{e, x, y\}), (Ϭ, \{e, x\})\}$ and $g_G = \{(Ձ, \{e, yx\}), (Ϭ, \{y\})\}$

Since $Ձ = ՁՁ = бб$, $(\mathscr{f}_G \otimes_{u/p} \mathscr{g}_G)(Ձ) = (\mathscr{f}_G{}^c(Ձ) \cup \mathscr{g}_G(Ձ)) \cup (\mathscr{f}_G{}^c(б) \cup \mathscr{g}_G(б)) = \{e, y, yx\}$ and since $б = Ձб = бՁ$, $(\mathscr{f}_G \otimes_{u/p} \mathscr{g}_G)(б) = (\mathscr{f}_G{}^c(Ձ) \cup \mathscr{g}_G(б)) \cup (\mathscr{f}_G{}^c(б) \cup \mathscr{g}_G(Ձ)) = \{e, y, yx\}$ is obtained. Hence,

$$\mathscr{f}_G \otimes_{u/p} \mathscr{g}_G = \{(Ձ, \{e, y, yx\}), (б, \{e, y, yx\})\}$$

Proposition 3.4. The set $S_G(U)$ is closed under the soft union-plus product. That is, if $\mathscr{f}_G$ and $\mathscr{g}_G$ are two $\mathcal{SS}$s, then so is $\mathscr{f}_G \otimes_{u/p} \mathscr{g}_G$.

PROOF. It is obvious that the soft union-plus product is a binary operation in $S_G(U)$. Thereby, $S_G(U)$ is closed under the soft union-plus product.

Proposition 3.5. The soft union-plus product is not associative in $S_G(U)$.

PROOF. Consider the group G and the $\mathcal{SS}$s $\mathscr{f}_G$ and $\mathscr{g}_G$ in Example 3.3. Let $\hbar_G$ be an $\mathcal{SS}$s over $U = \{e, x, y, yx\}$ such that $\hbar_G = \{(Ձ, \{x\}), (б, \{y\})\}$. Since $\mathscr{f}_G \otimes_{u/p} \mathscr{g}_G = \{(Ձ, \{e, y, yx\}), (б, \{e, y, yx\})\}$, then

$$(\mathscr{f}_G \otimes_{u/p} \mathscr{g}_G) \otimes_{u/p} \hbar_G = \{(Ձ, \{x, y\}), (б, \{x, y\})\}$$

Moreover, since $\mathscr{g}_G \otimes_{u/p} \hbar_G = \{(Ձ, U), (б, U)\}$, then

$$\mathscr{f}_G \otimes_{u/p} (\mathscr{g}_G \otimes_{u/p} \hbar_G) = \{(Ձ, U), (б, U)\}$$

Thereby, $(\mathscr{f}_G \otimes_{u/p} \mathscr{g}_G) \otimes_{u/p} \hbar_G \neq \mathscr{f}_G \otimes_{u/p} (\mathscr{g}_G \otimes_{u/p} \hbar_G)$.

Proposition 3.6. The soft union-plus product is not commutative in $S_G(U)$.

PROOF. Consider the $\mathcal{SS}$s $\mathscr{f}_G$ and $\mathscr{g}_G$ over $U = \{e, x, y, yx\}$ in Example 3.3. Then,

$$\mathscr{f}_G \otimes_{u/p} \mathscr{g}_G = \{(Ձ, \{e, y, yx\}), (б, \{e, y, yx\})\}, \text{ and } \mathscr{g}_G \otimes_{u/p} \mathscr{f}_G = \{(Ձ, U), (б, U)\}$$

implying that $\mathscr{f}_G \otimes_{u/p} \mathscr{g}_G \neq \mathscr{g}_G \otimes_{u/p} \mathscr{f}_G$.

Proposition 3.7. The soft union-plus product is not idempotent in $S_G(U)$.

PROOF. Consider the $\mathcal{SS}$ $\mathscr{f}_G$ in Example 3.3. Then, for all $x \in G$,

$$\mathscr{f}_G \otimes_{u/p} \mathscr{f}_G = \{(Ձ, U), (б, U)\}$$

implying that $\mathscr{f}_G \otimes_{u/p} \mathscr{f}_G \neq \mathscr{f}_G$. □

Proposition 3.8. Let $\mathscr{f}_G$ be a constant $\mathcal{SS}$. Then, $\mathscr{f}_G \otimes_{u/p} \mathscr{f}_G = U_G$.

PROOF. Let $\mathscr{f}_G$ be a constant $\mathcal{SS}$ such that, for all $x \in G$, $\mathscr{f}_G(x) = A$, where A is a fixed set. Hence, for all $x \in G$,

$$\begin{aligned}(\mathscr{f}_G \otimes_{u/p} \mathscr{f}_G)(x) &= \bigcup_{x=yz} (\mathscr{f}_G{}^c(y) \cup \mathscr{f}_G(z)) \\ &= U_G(x)\end{aligned}$$

Thereby, $\mathscr{f}_G \otimes_{u/p} \mathscr{f}_G = U_G$.□

Remark 3.9. Let $S_G{}^*(U)$ be the collection of all constant $\mathcal{SS}$s. Then, the soft union-plus product is not idempotent in $S_G{}^*(U)$ either.

Proposition 3.10. U_G is the right absorbing element of the soft union-plus product in $S_G(U)$.

PROOF. Let $x \in G$. Then, for all $x \in G$,

$$\begin{aligned}(\mathscr{f}_G \otimes_{u/p} U_G)(x) &= \bigcup_{x=yz} \left(\mathscr{f}_G{}^c(y) \cup U_G(z)\right) \\ &= \bigcup_{x=yz} \left(\mathscr{f}_G{}^c(y) \cup U\right) \\ &= U_G(x)\end{aligned}$$

Thus, $\mathscr{f}_G \otimes_{u/p} U_G = U_G$.□

Proposition 3.11. U_G is not the left absorbing element of the soft union-plus product in $S_G(U)$.

PROOF. Consider the $\mathcal{SS}$ $\mathscr{f}_G$ in Example 3.3. Then,

$$U_G \otimes_{u/p} \mathscr{f}_G = \{(\mathfrak{Q}, \{e, x, y\}), (\mathfrak{b}, \{e, x, y\})\}$$

implying that $U_G \otimes_{u/p} \mathscr{f}_G \neq U_G$.□

Remark 3.12. U_G is not the absorbing element of the soft union-plus product in $S_G(U)$.

Proposition 3.13. Let $\mathscr{f}_G$ be a constant $\mathcal{SS}$. Then, $U_G \otimes_{u/p} \mathscr{f}_G = \mathscr{f}_G$.

PROOF. Let $\mathscr{f}_G$ be a constant $\mathcal{SS}$ such that, for all $x \in G$, $\mathscr{f}_G(x) = A$, where A is a fixed set. Hence, for all $x \in G$,

$$\begin{aligned}(U_G \otimes_{u/p} \mathscr{f}_G)(x) &= \bigcup_{x=yz} \left(U_G{}^c(y) \cup \mathscr{f}_G(z)\right) \\ &= \bigcup_{x=yz} \left(\emptyset \cup \mathscr{f}_G(z)\right) \\ &= \mathscr{f}_G(x)\end{aligned}$$

Thereby, $U_G \otimes_{u/p} \mathscr{f}_G = \mathscr{f}_G$.□

Remark 3.14. U_G is the left identity element of the soft union-plus product in $S_G{}^*(U)$ by Proposition 3.10 and Proposition 3.13.

Proposition 3.15. Let $\mathscr{f}_G$ be an $\mathcal{SS}$. Then, $\emptyset_G \otimes_{u/p} \mathscr{f}_G = U_G$.

PROOF. Let $\mathscr{f}_G$ be an $\mathcal{SS}$. Then, for all $x \in G$,

$$\begin{aligned}(\emptyset_G \otimes_{u/p} \mathscr{f}_G)(x) &= \bigcup_{x=yz} \left(\emptyset_G{}^c(y) \cup \mathscr{f}_G(z)\right) \\ &= \bigcup_{x=yz} \left(U \cup \mathscr{f}_G(z)\right) \\ &= U_G(x)\end{aligned}$$

Thereby, $\emptyset_G \otimes_{u/p} \mathscr{f}_G = U_G$.□

Proposition 3.16. Let $\mathscr{f}_G$ be a constant $\mathcal{SS}$. Then, $\mathscr{f}_G \otimes_{u/p} \emptyset_G = \mathscr{f}_G{}^c$.

PROOF. Let $\mathscr{f}_G$ be a constant $\mathcal{SS}$ such that, for all $x \in G$, $\mathscr{f}_G(x) = A$, where A is a fixed set. Hence, for all $x \in G$,

$$\begin{aligned}(\mathscr{f}_G \otimes_{u/p} \emptyset_G)(x) &= \bigcup_{x=yz} \left(\mathscr{f}_G{}^c(y) \cup \emptyset_G(z)\right) \\ &= \bigcup_{x=yz} \left(\mathscr{f}_G{}^c(y) \cup \emptyset_G\right) \\ &= \mathscr{f}_G{}^c(x)\end{aligned}$$

Thereby, $\mathscr{f}_G \otimes_{u/p} \emptyset_G = \mathscr{f}_G{}^c$.

Proposition 3.17. Let $\mathscr{f}_G$ be a constant $\mathcal{SS}$. Then, ${\mathscr{f}_G}^c \otimes_{u/p} \mathscr{f}_G = \mathscr{f}_G$.

PROOF. Let $\mathscr{f}_G$ be a constant $\mathcal{SS}$ such that, for all $x \in G$, $\mathscr{f}_G\,(x) = A$, where A is a fixed set. Hence, for all $x \in G$,

$$\begin{aligned}({\mathscr{f}_G}^c \otimes_{u/p} \mathscr{f}_G)(x) &= \bigcup_{x=yz} \left(({\mathscr{f}_G}^c)^c(y) \cup \mathscr{f}_G(z) \right) \\ &= \bigcup_{x=yz} \left(\mathscr{f}_G(y) \cup \mathscr{f}_G(z) \right) \\ &= \mathscr{f}_G(x)\end{aligned}$$

Thereby, ${\mathscr{f}_G}^c \otimes_{u/p} \mathscr{f}_G = \mathscr{f}_G$.□

Proposition 3.18. Let $\mathscr{f}_G$ be a constant $\mathcal{SS}$. Then, $\mathscr{f}_G \otimes_{u/p} {\mathscr{f}_G}^c = {\mathscr{f}_G}^c$.

PROOF. Let $\mathscr{f}_G$ be a constant $\mathcal{SS}$ such that, for all $x \in G$, $\mathscr{f}_G\,(x) = A$, where A is a fixed set. Hence, for all $x \in G$,

$$(\mathscr{f}_G \otimes_{u/p} {\mathscr{f}_G}^c)(x) = \bigcup_{x=yz} \left({\mathscr{f}_G}^c(y) \cup {\mathscr{f}_G}^c(z) \right) = {\mathscr{f}_G}^c(x)$$

Thereby, $\mathscr{f}_G \otimes_{u/p} {\mathscr{f}_G}^c = {\mathscr{f}_G}^c$.□

Proposition 3.19. Let $\mathscr{f}_G$ and $\mathscr{g}_G$ be two $\mathcal{SS}$s. Then, $\mathscr{f}_G \otimes_{u/p} \mathscr{g}_G = \emptyset_G$ if and only if $\mathscr{f}_G = U_G$ and $\mathscr{g}_G = \emptyset_G$.

PROOF. Let $\mathscr{f}_G$ and $\mathscr{g}_G$ be two $\mathcal{SS}$s. Suppose that $\mathscr{f}_G = U_G$ and $\mathscr{g}_G = \emptyset_G$. Then, $\mathscr{f}_G(x) = U_G(x) = U$ and $\mathscr{g}_G(x) = \emptyset_G(x) = \emptyset$, for all $x \in G$. Thus, for all $x \in G$,

$$(\mathscr{f}_G \otimes_{u/p} \mathscr{g}_G)(x) = \bigcup_{x=yz} \left({\mathscr{f}_G}^c(y) \cup \mathscr{g}_G(z) \right) = \emptyset = \emptyset_G(x)$$

Thereby, $\mathscr{f}_G \otimes_{u/p} \mathscr{g}_G = \emptyset_G$.

Conversely, suppose that $\mathscr{f}_G \otimes_{u/p} \mathscr{g}_G = \emptyset_G$. Then, $(\mathscr{f}_G \otimes_{u/p} \mathscr{g}_G)(x) = \emptyset_G(x) = \emptyset$ for all $x \in G$. Thus, for all $x \in G$,

$$\emptyset_G(x) = \emptyset = (\mathscr{f}_G \otimes_{u/p} \mathscr{g}_G)(x) = \bigcup_{x=yz} \left({\mathscr{f}_G}^c(y) \cup \mathscr{g}_G(z) \right)$$

This implies that ${\mathscr{f}_G}^c(y) \cup \mathscr{g}_G(z) = \emptyset$, for all $y, z \in G$. Thus, $\mathscr{f}_G(x) = U_G(x) = U$ and $\mathscr{g}_G(x) = \emptyset_G(x) = \emptyset$, for all $x \in G$. Thereby, $\mathscr{f}_G = U_G$ and $\mathscr{g}_G = \emptyset_G$. □

Proposition 3.20. Let $\mathscr{f}_G$ and $\mathscr{g}_G$ be two $\mathcal{SS}$s. If one of the following assertions is satisfied, then $\mathscr{f}_G \otimes_{u/p} \mathscr{g}_G = U_G$:

i. $\mathscr{f}_G \subseteq_A \mathscr{g}_G$
ii. $\mathscr{f}_G =_S \mathscr{g}_G$
iii. $\mathscr{g}_G = U_G$
iv. $\mathscr{f}_G = \emptyset_G$

PROOF. Let $\mathscr{f}_G$ and $\mathscr{g}_G$ be two $\mathcal{SS}$s over U.

i. Let $\mathscr{f}_G \subseteq_A \mathscr{g}_G$. Then, for each $x, y \in G$, $\mathscr{f}_G(x) \subseteq \mathscr{g}_G(y)$. Thus, for all $x \in G$,

$$(\mathscr{f}_G \otimes_{u/p} \mathscr{g}_G)(x) = \bigcup_{x=yz} \left({\mathscr{f}_G}^c(y) \cup \mathscr{g}_G(z) \right) = U_G$$

Note here that, ${\mathscr{f}_G}^c(y) \cup \mathscr{g}_G(z) = \left(\mathscr{f}_G\,(y) \setminus \mathscr{g}_G(z) \right)'$, for all $y, z \in G$.

ii. It follows by Proposition 3.8.

iii. It follows by Proposition 3.10.

iv. It follows by Proposition 3.15.

Proposition 3.21. Let f_G and g_G be two SSs. If $g_G \,\tilde{\subseteq}_S\, (f_G)^c$, then $f_G \otimes_{u/p} g_G = f_G{}^c$.

PROOF. Let f_G and g_G be two SSs and $g_G \,\tilde{\subseteq}_S\, (f_G)^c$. Hence, for all $x \in G$, $f_G(x) = A$ and $g_G(x) = B$, where A and B are two fixed sets and $B \subseteq A'$. Thus, for all $x \in G$,

$$(f_G \otimes_{u/p} g_G)(x) = \bigcup_{x=yz} \left(f_G{}^c(y) \cup g_G(z)\right) = f_G{}^c(x)$$

Thereby, $f_G \otimes_{u/p} g_G = f_G{}^c$. □

Proposition 3.22. Let f_G and g_G be two SSs. If $f_G{}^c \,\tilde{\subseteq}_S\, g_G$, then $f_G \otimes_{u/p} g_G = g_G$.

PROOF. Let f_G and g_G be two SSs and $f_G{}^c \,\tilde{\subseteq}_S\, g_G$. Hence, for all $x \in G$, $f_G(x) = A$ and $g_G(x) = B$, where A and B are two fixed sets and $A' \subseteq B$. Thus, for all $x \in G$,

$$(f_G \otimes_{u/p} g_G)(x) = \bigcup_{x=yz} \left(f_G{}^c(y) \cup g_G(z)\right) = g_G(x)$$

Thereby, $f_G \otimes_{u/p} g_G = g_G$. □

Proposition 3.23. Let f_G and g_G be two SSs. Then, $\left(f_G \otimes_{u/p} g_G\right)^c = f_G \otimes_{i/d} g_G$.

PROOF. Let f_G and g_G be two SSs. Then, for all $x \in G$,

$$\begin{aligned}
\left(f_G \otimes_{u/p} g_G\right)^c(x) &= \left(\bigcup_{x=yz} \left(f_G{}^c(y) \cup g_G(z)\right)\right)' \\
&= \bigcap_{x=yz} \left(f_G{}^c(y) \cup g_G(z)\right)' \\
&= \bigcap_{x=yz} \left(f_G(y) \cap g_G{}^c(z)\right) \\
&= \bigcap_{x=yz} \left(f_G(y) \setminus g_G(z)\right) \\
&= (f_G \otimes_{i/d} g_G)(x)
\end{aligned}$$

Thereby, $\left(f_G \otimes_{u/p} g_G\right)^c = f_G \otimes_{i/d} g_G$.

Proposition 3.24. Let f_G, g_G, and $\hbar_G$ be three SSs. If $f_G \,\tilde{\subseteq}\, g_G$, then $g_G \otimes_{u/p} \hbar_G \,\tilde{\subseteq}\, f_G \otimes_{u/p} \hbar_G$ and $\hbar_G \otimes_{u/p} f_G \,\tilde{\subseteq}\, \hbar_G \otimes_{u/p} g_G$.

PROOF. Let f_G, g_G, and $\hbar_G$ be three SSs such that $f_G \,\tilde{\subseteq}\, g_G$. Then, for all $x \in G$, $f_G(x) \subseteq g_G(x)$, and hence, $g_G{}^c(x) \subseteq f_G{}^c(x)$. Then, for all $x \in G$,

$$\begin{aligned}
\left(g_G \otimes_{u/p} \hbar_G\right)(x) &= \bigcup_{x=yz} \left(g_G{}^c(y) \cup \hbar_G(z)\right) \\
&\subseteq \bigcup_{x=yz} \left(f_G{}^c(y) \cup \hbar_G(z)\right) \\
&= \left(f_G \otimes_{u/p} \hbar_G\right)(x)
\end{aligned}$$

implying that $g_G \otimes_{u/p} \hbar_G \,\tilde{\subseteq}\, f_G \otimes_{u/p} \hbar_G$. Similarly, for all $x \in G$,

$$\begin{aligned}(\hbar_G \otimes_{u/p} f_G)(x) &= \bigcup_{x=yz} \left(\hbar_G{}^c(y) \cup f_G(z)\right)\\ &\subseteq \bigcup_{x=yz} \left(\hbar_G{}^c(y) \cup g_G(z)\right)\\ &= (\hbar_G \otimes_{u/p} g_G)(x)\end{aligned}$$

implying that $\hbar_G \otimes_{u/p} f_G \tilde{\subseteq} \hbar_G \otimes_{u/p} g_G$.□

Proposition 3.25. Let f_G, g_G, σ_G, and k_G be four SSs. If $k_G \tilde{\subseteq} \sigma_G$, and $f_G \tilde{\subseteq} g_G$, then $\sigma_G \otimes_{u/p} f_G \tilde{\subseteq} k_G \otimes_{u/p} g_G$ and $g_G \otimes_{u/p} k_G \tilde{\subseteq} f_G \otimes_{u/p} \sigma_G$.

PROOF. Let f_G, g_G, σ_G, and k_G be four SSs such that $k_G \tilde{\subseteq} \sigma_G$, and $f_G \tilde{\subseteq} g_G$. Then, for all $x \in G$, $k_G(x) \subseteq \sigma_G(x)$, $f_G(x) \subseteq g_G(x)$, and thus, $\sigma_G{}^c(x) \subseteq k_G{}^c(x)$, $g_G{}^c(x) \subseteq f_G{}^c(x)$. Then, for all $x \in G$,

$$\begin{aligned}(\sigma_G \otimes_{u/p} f_G)(x) &= \bigcup_{x=yz} \left(\sigma_G{}^c(y) \cup f_G(z)\right)\\ &\subseteq \bigcup_{x=yz} \left(k_G{}^c(y) \cup g_G(z)\right)\\ &= (k_G \otimes_{u/p} g_G)(x)\end{aligned}$$

implying that $\sigma_G \otimes_{u/p} f_G \tilde{\subseteq} k_G \otimes_{u/p} g_G$. Similarly, for all $x \in G$,

$$\begin{aligned}(g_G \otimes_{u/p} k_G)(x) &= \bigcup_{x=yz} \left(g_G{}^c(y) \cup k_G(z)\right)\\ &\subseteq \bigcup_{x=yz} \left(f_G{}^c(y) \cup \sigma_G(z)\right)\\ &= (f_G \otimes_{u/p} \sigma_G)(x)\end{aligned}$$

implying that $g_G \otimes_{u/p} k_G \tilde{\subseteq} f_G \otimes_{u/p} \sigma_G$. □

Proposition 3.26. The soft union-plus product distributes over the union operation of SSs from the left side.

PROOF. Let f_G, g_G, and $\hbar_G$ be three SSs. Then, for all $x \in G$,

$$\begin{aligned}\left(f_G \otimes_{u/p} (g_G \tilde{\cup} \hbar_G)\right)(x) &= \bigcup_{x=yz} \left(f_G{}^c(y) \cup (g_G \tilde{\cup} \hbar_G)(z)\right)\\ &= \bigcup_{x=yz} \left(f_G{}^c(y) \cup \left(g_G(z) \cup \hbar_G(z)\right)\right)\\ &= \bigcup_{x=yz} \left(\left(f_G{}^c(y) \cup g_G(z)\right) \cup \left(f_G{}^c(y) \cup \hbar_G(z)\right)\right)\\ &= \left[\bigcup_{x=yz} \left(f_G{}^c(y) \cup g_G(z)\right)\right] \cup \left[\bigcup_{x=yz} \left(f_G{}^c(y) \cup \hbar_G(z)\right)\right]\\ &= (f_G \otimes_{u/p} g_G)(x) \cup (f_G \otimes_{u/p} \hbar_G)(x)\end{aligned}$$

Thus, $f_G \otimes_{u/p} (g_G \tilde{\cup} \hbar_G) = (f_G \otimes_{u/p} g_G) \tilde{\cup} (f_G \otimes_{u/p} \hbar_G)$.□

Example 3.27. Consider the group G in Example 3.3. Let f_G, g_G, and $\hbar_G$ be three SSs over $U = \{e, x, y, yx\}$ as follows:

$$f_G = \{(\mathfrak{a}, \{e, x, y\}), (\mathfrak{b}, \{e, x\})\},\ g_G = \{(\mathfrak{a}, \{e, yx\}), (\mathfrak{b}, \{y\})\},\ \hbar_G = \{(\mathfrak{a}, \{x\}), (\mathfrak{b}, \{y\})\}$$

Since $f_G \otimes_{u/p} g_G = \{(\mathfrak{a}, \{e, y, yx\}), (\mathfrak{b}, \{e, y, yx\})\}$ and $f_G \otimes_{u/p} \hbar_G = \{(\mathfrak{a}, \{x, y, yx\}), (\mathfrak{b}, \{x, y, yx\})\}$, then

$$\left(f_G \otimes_{u/p} g_G\right) \tilde{\cup} \left(f_G \otimes_{u/p} \hbar_G\right) = \{(\text{Ձ}, U), (\text{ɓ}, U)\}$$

Moreover, since $g_G \tilde{\cup} \hbar_G = \{(\text{Ձ}, \{e, x, yx\}), (\text{ɓ}, \{y\})\}$

$$f_G \otimes_{u/p} (g_G \tilde{\cup} \hbar_G) = \{(\text{Ձ}, U), (\text{ɓ}, U)\}$$

Thus, $f_G \otimes_{u/p} (g_G \tilde{\cup} \hbar_G) = \left(f_G \otimes_{u/p} g_G\right) \tilde{\cup} \left(f_G \otimes_{u/p} \hbar_G\right)$.□

Proposition 3.28. The soft union-plus product does not distribute over the intersection operation of $\mathcal{SS}$s from the right side.

PROOF. Consider the group G in Example 3.3. Let f_G, g_G, and $\hbar_G$ be three $\mathcal{SS}$s over $U = \{e, x, y, yx\}$ as follows:

$$f_G = \{(\text{Ձ}, \{e, x, y\}), (\text{ɓ}, \{e, x\})\},\ g_G = \{(\text{Ձ}, \{e, yx\}), (\text{ɓ}, \{y\})\},\ \hbar_G = \{(\text{Ձ}, \{x\}), (\text{ɓ}, \{y\})\}$$

Since $f_G \otimes_{u/p} \hbar_G = \{(\text{Ձ}, \{x, y, yx\}), (\text{ɓ}, \{x, y, yx\})\}$ and $g_G \otimes_{u/p} \hbar_G = \{(\text{Ձ}, U), (\text{ɓ}, U)\}$, then

$$\left(f_G \otimes_{u/p} \hbar_G\right) \tilde{\cup} \left(g_G \otimes_{u/p} \hbar_G\right) = \{(\text{Ձ}, U), (\text{ɓ}, U)\}$$

Moreover, since $f_G \tilde{\cup} g_G = \{(\text{Ձ}, U), (\text{ɓ}, \{e, x, y\})\}$

$$(f_G \tilde{\cup} g_G) \otimes_{u/p} \hbar_G = \{(\text{Ձ}, \{x, y, yx\}), (\text{ɓ}, \{x, y, yx\})\}$$

Thus, $(f_G \tilde{\cup} g_G) \otimes_{u/p} \hbar_G \neq \left(f_G \otimes_{u/p} \hbar_G\right) \tilde{\cup} \left(g_G \otimes_{u/p} \hbar_G\right)$.□

Remark 3.29. The soft union-plus product does not distribute over the union operation of $\mathcal{SS}$s from both sides.

4. CONCLUSION

This study formally introduces a novel binary operation on soft sets—the soft union-plus product—defined over parameter domains equipped with intrinsic group-theoretic structures. Building on this formulation, a comprehensive algebraic analysis is conducted, focusing on the operation's behavior within various hierarchies of soft subsethood and its compatibility with generalized notions of soft equality. A rigorous comparative framework situates the proposed operation alongside existing binary soft products, offering sharpened insights into their relative expressive power and algebraic coherence. The analysis also explores the product's interactions with null and absolute soft sets, as well as with other group-based binary soft operations, thereby elucidating its structural role within the broader algebraic topology of soft set theory. Developed within a strictly axiomatic framework, the operation is examined with respect to key algebraic invariants—closure, associativity, commutativity, idempotency, distributivity, and the presence or absence of identity, inverse, and absorbing elements. The results confirm the internal consistency and formal robustness of the soft union-plus product, establishing it as a foundational tool for extending classical algebraic structures into the domain of soft sets. More broadly, the operation lays the groundwork for a generalized soft group theory, wherein soft sets over group-parameterized domains replicate core group-theoretic behaviors through rigorously defined soft operations. In addition to its theoretical significance, the proposed framework provides a rich basis for further algebraic developments and applications in abstract modeling, generalized soft equalities, and uncertainty-driven decision analysis.

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