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Distance and similarity measures on interval-valued Q-neutrosophic soft environments and their applications

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ABSTRACT: Decision-making problems are characterized by many issues of uncertainty and hiddenness, and this requires identifying and developing the mathematical tools used to deal with them more accurately. For this purpose, in this work we will present an innovative mathematical concept called interval valued-Q-neutrosophic soft sets (IV-Q-NSSs) as a hyper fuzzy extension of some previous models. To clarify this concept further, we will provide some illustrative examples. After that, we discovered the distance and similarity measures on IV-Q-NSSs. We proposed several types of these measures and illustrated them with several numerical examples. Based on these measures, we created an innovative algorithm based on these procedures to solve one of the problems of daily life.
Keywords: Fuzzy set; neutrosophic set; soft set; Q- neutrosophic set, Q- neutrosophic soft set

1. INTRODUCTION

Smarandache [1] in 1999 developed the idea of neutrosophic sets as a new point of view on the uncertainty and vagueness of data. This idea is considered to extend fuzzy sets (FSs) [2] and intuitionistic fuzzy sets (IFSs) [3]. By carefully examining the structure of this concept, we find that the mechanism of this concept is to give three values to each element present within the universal set. This idea was appreciated by many researchers around the world and prompted them to innovate many research works that contributed to addressing many problems of daily life i.e. In engineering, computer science, economics, business administration, and many fields of practical life. In addition to the advantages possessed by this concept, some drawbacks hinder the data processing process inherent in everyday problems. One of these problems is the inability of these concepts to deal with the mechanism of representing parameters associated with the data of the problem. To handle this problem, the concept of the soft sets theory was introduced by Molodtsov [4] in (1999) as a new parametric family that has the flexible ability to handle different design-making issues. After that, the above concepts were combined with SS by Karaaslan 2015, Maji 2013 when they proposed neutrosophic soft sets [5] as an extension of fuzzy soft sets [6] and intuitionistic fuzzy soft sets [7]. Deli [10] generalises the notions of SS and NS to interval-NSs under interval form. Saber et al. [11] started the research on the topological-NS information of soft sets by introducing a new approach called single-valued neutrosophic soft topological space. In complex spaces, a lot of research has been introduced [12-18].

In other side, the fuzzy set environment and its extension lack the ability to handle two-dimensional information that is available in universal discourse U. For example, if we consider that U contains three patients, u_1, u_2, and u_3, who are suspected of being infected with a disease, it is difficult to describe their condition through a single object (one dimension). This motivates Adam and Hassan [21] to propose new strategies when they build a new model of Q-fuzzy sets (Q-FSs) to serve uncertainty and two-dimensionality simultaneously. After that, Broumi [22] extended to a Q-intuitionistic fuzzy soft set by combining IFSs and SSs by adding a two-dimensional non-membership function. These models are an extension of FSs and IFSs, so it is not feasible to deal with uncertain information that is saturated with positions of neutrality and ambiguity. To address this aspect, recently Abu Qamar and Hassan [23] established the notion of Q-neutrosophic soft sets (Q-NSSs) as a generalisation of NSSs and Q-FSs by upgrading the membership functions of NSSs to two dimensions. This approach has good capabilities compared to the works mentioned in this literature, but the outputs of this model are single values. As we mentioned previously, these values constitute an obstacle for the decision-maker and do not give him sufficient freedom to build numerical data that describes the data of the problem to be solved.

Currently, distance and similarity measures are among the basic tools in the fuzzy environment because of their importance in finding convergence between the data determined by this environment i.e. it works to determine the degree of similarity or distance between two objects.
This manuscript aimed to suggest techniques a new idea called IV-Q-NSSs, which stands for interval-valued Q-neutrosophic soft sets. These are a more developed form of Q-NSSs, and each membership function is unique to Q-NSSs given in interval form. This format gives the user more freedom and efficiency when dealing with everyday scenarios, especially those saturated with neutral, two-dimensional uncertainty information

The main contributions shown in this work that were made to achieve these objectives are:

i. A new technique (IV-Q-NSSs) is proposed to contain the effects of uncertainty information in two-dimensional.
ii. To demonstrate the distance and similarity measures between two IV-Q-NSSs and supported them by an illustrative numerical example.
iii. On the applied side, these techniques have been added to solve one of the decision-making problems in the medical field by proposing a multi-step algorithm that works on distance and similarity measures between two IV-Q-NSSs data.

The following diagram presents the rest of the paper:

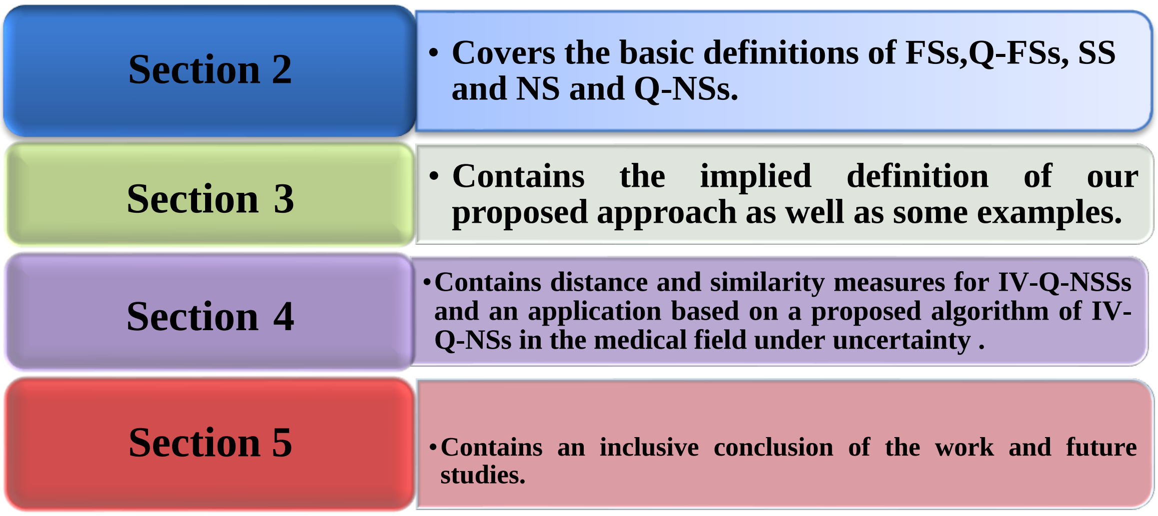

Figure 1: a representation of results.

2. PRELIMINARIES

In this part, we recollect some critical notions related to our proposed approach like FS, Q-FS, SS, and NS.

Definition 2.1. [1] Assume that $\mathfrak{U} = \{\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{u}_3, \ldots, \mathfrak{u}_n\}$ be the initial points space(non-empty universal set). Then an FS $\mathcal{F}$ on $\mathfrak{U}$ is defined by following form:

$$\mathcal{F} = \{\mathfrak{u}_j, \hat{P}^t(\mathfrak{u}_j) | \mathfrak{u}_j \in \mathfrak{U}\}$$

Where $\mathcal{F}$ is a mapping defined as $\mathcal{F}: \mathfrak{U} \to [0,1]$ such that $\hat{P}^t \in [0,1]$ and called truth membership function (TMF).

Definition 2.2. [21] Assume that $\mathfrak{U} = \{\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{u}_3, \dots, \mathfrak{u}_n\}$ be the initial points space(non-empty universal set) and $\mathfrak{Q} = \{\mathfrak{q}_1, \mathfrak{q}_2, \mathfrak{q}_3, \dots, \mathfrak{q}_n\}$ be nonempty set. Then an Q-FS $\mathcal{F}_{\mathfrak{Q}}$ on the order pair $(\mathfrak{U}, \mathfrak{Q})$is defined by following form:

$$\mathcal{F}_{\mathfrak{Q}} = \{(u, \mathfrak{q}), \hat{P}^t(u, \mathfrak{q}) | (u, \mathfrak{q}) \in \mathfrak{U} \times \mathfrak{Q}\}$$

Where $\mathcal{F}$ is a mapping defined as $\mathcal{F}_{\mathfrak{Q}}: \mathfrak{U} \times \mathfrak{Q} \to [0,1]$ such that $\hat{P}^t_{\mathfrak{Q}} \in [0,1]$ and called Q-truth membership function (TMF).

Definition 2.3. [3] Assume that $\mathfrak{U} = \{\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{u}_3, \dots, \mathfrak{u}_n\}$ be the initial points space(non-empty universal set). Then an NS N on $\mathfrak{U}$ is defined by following form:

$$\mathrm{N} = \{\mathfrak{u}_j, \hat{P}^t(\mathfrak{u}_j), \hat{P}^i(\mathfrak{u}_j), \hat{P}^f(\mathfrak{u}_j) | \mathfrak{u}_j \in \mathfrak{U}\}$$

Where N is a mapping defined as $\mathrm{N}: \mathfrak{U} \to [0,1]$ such that $\hat{P}^t(\mathfrak{u}_j), \hat{P}^i(\mathfrak{u}_j), \hat{P}^f(\mathfrak{u}_j) \in [0,1]$ and called truth membership function (TMF), neutrality membership function (NMF), and falsity membership function (FMF) with stander condition $0 \le \hat{P}^t(\mathfrak{u}_j) + \hat{P}^i(\mathfrak{u}_j) + \hat{P}^f(\mathfrak{u}_j) \le 1$.

Definition 2.4. [24] Assume that $\mathfrak{U} = \{\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{u}_3, \dots, \mathfrak{u}_n\}$ be the initial points space(non-empty universal set). Then an Q-NS N on $(\mathfrak{U} \times \mathfrak{Q})$is defined by following form:

$$\mathrm{N}_{\mathfrak{Q}} = \{\mathfrak{u}_j, \hat{P}^t_{\mathfrak{Q}}(u, \mathfrak{q}), \hat{P}^{\mathrm{i}}_{\mathfrak{Q}}(u, \mathfrak{q}), \hat{P}^{\mathfrak{f}}_{\mathfrak{Q}}(u, \mathfrak{q}) | (u, \mathfrak{q}) \in \mathfrak{U} \times \mathfrak{Q}\}$$

Where $\mathrm{N}_{\mathfrak{Q}}$ is a mapping defined as $\mathrm{N}_{\mathfrak{Q}}: \mathfrak{U} \times \mathfrak{Q} \to [0,1]$ such that $\hat{P}^t_{\mathfrak{Q}}(u, \mathfrak{q}), \hat{P}^{\mathrm{i}}_{\mathfrak{Q}}(u, \mathfrak{q}), \hat{P}^{\mathfrak{f}}_{\mathfrak{Q}}(u, \mathfrak{q}) \in [0,1]$ and called truth membership function (TMF), neutrality membership function (NMF), and falsity membership function (FMF) with stander condition $0 \le \hat{P}^t_{\mathfrak{Q}}(u, \mathfrak{q}) + \hat{P}^{\mathrm{i}}_{\mathfrak{Q}}(u, \mathfrak{q}) + \hat{P}^{\mathfrak{f}}_{\mathfrak{Q}}(u, \mathfrak{q}) \le 1$.

Definition 2.5. [10] Assume that $\mathfrak{U} = \{\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{u}_3, \dots, \mathfrak{u}_n\}$ be the initial points space(non-empty universal set). Then an IVNS N on $\mathfrak{U}$ is defined by following form:

$$\mathrm{N} = \{\mathfrak{u}_j, \hat{P}^t(\mathfrak{u}_j), \hat{P}^i(\mathfrak{u}_j), \hat{P}^f(\mathfrak{u}_j) | \mathfrak{u}_j \in \mathfrak{U}\}$$

Where $\hat{P}^t(\mathfrak{u}_j) = [\hat{P}^{t,l}(\mathfrak{u}_j), \hat{P}^{t,u}(\mathfrak{u}_j)]$, $\hat{P}^i(\mathfrak{u}_j) = [\hat{P}^{i,l}(\mathfrak{u}_j), \hat{P}^{i,u}(\mathfrak{u}_j)]$ and $\hat{P}^f(\mathfrak{u}_j) = [\hat{P}^{f,l}(\mathfrak{u}_j), \hat{P}^{f,u}(\mathfrak{u}_j)]$
Such that the domen of these terms is $\mathfrak{U}$ and the co-domen is $[0,1]$ and $\hat{P}^{t,l}(\mathfrak{u}_j), \hat{P}^{t,u}(\mathfrak{u}_j)$ are lower and upper of TMF, $\hat{P}^{i,l}(\mathfrak{u}_j), \hat{P}^{i,u}(\mathfrak{u}_j)$ are lower and upper of IMF and $\hat{P}^{f,l}(\mathfrak{u}_j), \hat{P}^{f,u}(\mathfrak{u}_j)$ are lower and upper of FMF, with two stander conditions $0 \le \hat{P}^{t,l}(\mathfrak{u}_j) + \hat{P}^{i,l}(\mathfrak{u}_j) + \hat{P}^{f,l}(\mathfrak{u}_j) \le 1$ and $0 \le \hat{P}^{t.u}(\mathfrak{u}_j) + \hat{P}^{i,u}(\mathfrak{u}_j) + \hat{P}^{f,u}(\mathfrak{u}_j) \le 1$.

Definition 2.6. [10] Assume that
$N_1 = \{\mathfrak{u}_j, \hat{P}^t_1(\mathfrak{u}_j), \hat{P}^i_1(\mathfrak{u}_j), \hat{P}^f_1(\mathfrak{u}_j) | \mathfrak{u}_j \in \mathfrak{U}\}$, $N_2 = \{\mathfrak{u}_j, \hat{P}^t_2(\mathfrak{u}_j), \hat{P}^i_2(\mathfrak{u}_j), \hat{P}^f_2(\mathfrak{u}_j) | \mathfrak{u}_j \in \mathfrak{U}\}$ be two INS on initial points space(non-empty universal set) $\mathfrak{U}$
where $\hat{P}^t_1(\mathfrak{u}_j) = [\hat{P}^{t,l}_1(\mathfrak{u}_j), \hat{P}^{t,u}_1(\mathfrak{u}_j)]$, $\hat{P}^i_1(\mathfrak{u}_j) = [\hat{P}^{i,l}_1(\mathfrak{u}_j), \hat{P}^{i,u}_1(\mathfrak{u}_j)]$ and $\hat{P}^f_1(\mathfrak{u}_j) = [\hat{P}^{f,l}_1(\mathfrak{u}_j), \hat{P}^{f,u}_1(\mathfrak{u}_j)]$ and $\hat{P}^t_2(\mathfrak{u}_j) = [\hat{P}^{t,l}_2(\mathfrak{u}_j), \hat{P}^{t,u}_2(\mathfrak{u}_j)]$, $\hat{P}^i_2(\mathfrak{u}_j) = [\hat{P}^{i,l}_2(\mathfrak{u}_j), \hat{P}^{i,u}_2(\mathfrak{u}_j)]$ and $\hat{P}^f_2(\mathfrak{u}_j) = [\hat{P}^{f,l}_2(\mathfrak{u}_j), \hat{P}^{f,u}_2(\mathfrak{u}_j)]$ Then,

i. Complement $N_1^C = \{\mathfrak{u}_j, \hat{P}^f_1(\mathfrak{u}_j), 1 - \hat{P}^i_1(\mathfrak{u}_j), \hat{P}^t_1(\mathfrak{u}_j) | \mathfrak{u}_j \in \mathfrak{U}\}$

ii. Union: $N_1 \cup N_2 = \{\mathfrak{u}_j, max[\hat{P}^t_1(\mathfrak{u}_j), \hat{P}^t_2(\mathfrak{u}_j)], \min[\hat{P}^i_1(\mathfrak{u}_j), \hat{P}^i_2(\mathfrak{u}_j)], \min[\hat{P}^f_1(\mathfrak{u}_j), \hat{P}^f_2(\mathfrak{u}_j)] | \mathfrak{u}_j \in \mathfrak{U}\}$.

iii. Intersection: $N_1 \cap N_2 = \{\mathfrak{u}_j, min[\hat{P}^t_1(\mathfrak{u}_j), \hat{P}^t_2(\mathfrak{u}_j)], \max[\hat{P}^i_1(\mathfrak{u}_j), \hat{P}^i_2(\mathfrak{u}_j)], \max[\hat{P}^f_1(\mathfrak{u}_j), \hat{P}^f_2(\mathfrak{u}_j)] | \mathfrak{u}_j \in \mathfrak{U}\}$.

iv. Subset $N_1 \subseteq N_2$ if $\hat{P}^t_1(\mathfrak{u}_j) \le \hat{P}^t_2(\mathfrak{u}_j)$, $\hat{P}^i_1(\mathfrak{u}_j) \ge \hat{P}^i_2(\mathfrak{u}_j)$, $\hat{P}^f_1(\mathfrak{u}_j) \ge \hat{P}^f_2(\mathfrak{u}_j)$.

Definition 2.7. [7] A pair $(\mathcal{F}, \overline{\mathrm{A}} \subseteq \mathcal{E})$ is named SSs over a non-empty universe of discourse $\mathfrak{U}$ if $\mathcal{F}: \overline{\mathrm{A}} \subseteq \mathcal{E} \longrightarrow P(\mathfrak{U})$, such that the term $P(\mathfrak{U})$ indicate the power set of $\mathfrak{U}$.

3. THE MATHEMATICAL STRUCTURE OF INTERVAL VALUED-Q-NEUTROSOPHIC SOFT SETS (IV-Q-NSSS)

This section proposes the general framework definition of our concept IV-Q-NSS with fundamental operations like empty ICNSS, absolute ICNSS, subset ICNSS, and equality between two ICNSS. Also, to clarify our model more, we will give some numerical examples.

Definition 3.1. Assume that $\mathfrak{U} = \{\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{u}_3, \dots, \mathfrak{u}_n\}$ be the initial points space(non-empty universal set), $\mathfrak{Q} \ne \emptyset, ie\ \ \mathfrak{Q} = \{\mathfrak{q}_1, \mathfrak{q}_1, \mathfrak{q}_1, \dots, \mathfrak{q}_n\}$ and $\mathcal{E} = \{e_1, e_2, e_3, \dots, e_n\}$ be a set of attribute (parameters set). Let $\overline{\mathrm{A}} \subseteq \mathcal{E}$

be sub set of attribute set , then a duet $(\hat{P}_{\mathfrak{Q}}, \overline{\mathrm{A}})$ is called a interval-valued $\mathfrak{Q}$-neutrosophic soft set over the initial points space (non-empty universal set) $\mathfrak{U}$, where $\hat{P}_{\mathfrak{Q}}$ given as following mapping

$$\hat{P}_{\mathfrak{Q}}: \overline{\mathrm{A}} \to \mathfrak{Q} - IVNS(\mathfrak{U})$$

Then ,the $IV - \mathfrak{Q} - NSS\mathfrak{U})$ can be characterized by the following get form

$$(\hat{P}_{\mathfrak{Q}}, \overline{\mathrm{A}}) = \hat{P}_{\mathfrak{Q}_{\overline{\mathrm{A}}}} = \{e \in \overline{\mathrm{A}}, < \hat{P}_{\mathfrak{Q}}^{t}(u,\mathfrak{q})(e), \hat{P}_{\mathfrak{Q}}^{\mathrm{i}}(u,\mathfrak{q})(e), \hat{P}_{\mathfrak{Q}}^{\mathrm{f}}(u,\mathfrak{q})(e) > |(u,\mathfrak{q}) \in \mathfrak{U} \times \mathfrak{Q}\}$$

Where

$$\hat{P}_{\mathfrak{Q}}^{t}(u,\mathfrak{q})(\mathrm{e}) = [\hat{P}_{\mathfrak{Q}}^{t,l}(u,\mathfrak{q})(e), \hat{P}_{\mathfrak{Q}}^{t,u}(u,\mathfrak{q})(\mathrm{e})]$$
$$\hat{P}_{\mathfrak{Q}}^{i}(u,\mathfrak{q})(\mathrm{e}) = [\hat{P}_{\mathfrak{Q}}^{i,l}(u,\mathfrak{q})(e), \hat{P}_{\mathfrak{Q}}^{i,u}(u,\mathfrak{q})(\mathrm{e})]$$
$$\hat{P}_{\mathfrak{Q}}^{f}(u,\mathfrak{q})(\mathrm{e}) = [\hat{P}_{\mathfrak{Q}}^{f,l}(u,\mathfrak{q})(\mathrm{e}), \hat{P}_{\mathfrak{Q}}^{f,u}(u,\mathfrak{q})(\mathrm{e})]$$

Such that , the terms here $\hat{P}_{\mathfrak{Q}}^{t,l}(u,\mathfrak{q})(\mathrm{e})$, $\hat{P}_{\mathfrak{Q}}^{t,u}(u,\mathfrak{q})(\mathrm{e})$, $\hat{P}_{\mathfrak{Q}}^{i,l}(u,\mathfrak{q})(\mathrm{e})$, $\hat{P}_{\mathfrak{Q}}^{i,u}(u,\mathfrak{q})(\mathrm{e})$and $\hat{P}_{\mathfrak{Q}}^{f,l}(u,\mathfrak{q})(\mathrm{e})$, $\hat{P}_{\mathfrak{Q}}^{f,u}(u,\mathfrak{q})(\mathrm{e})$ refer to true interval-valued membership function, indeterminacy interval-valued membership function, and falsehood interval-valued membership function of objects $(u,\mathfrak{q}) \in \mathfrak{U} \times \mathfrak{Q}$, with two stander conditions $0 \leq \hat{P}_{\mathfrak{Q}}^{t,l}(u,\mathfrak{q})(e) + \hat{P}_{\mathfrak{Q}}^{i,l}(u,\mathfrak{q})(e) + \hat{P}_{\mathfrak{Q}}^{i,l}(u,\mathfrak{q})(e) \leq 1$ and $0 \leq \hat{P}_{\mathfrak{Q}}^{t,u}(u,\mathfrak{q})(\mathrm{e}) + \hat{P}_{\mathfrak{Q}}^{i,u}(u,\mathfrak{q})(\mathrm{e}) + \hat{P}_{\mathfrak{Q}}^{i,u}(u,\mathfrak{q})(\mathrm{e}) \leq 1$.

Now, to shed more light on the above definition, we present below the following numerical example, which describes the mechanism of action of our approach presented in this work.

Example 3.2. Assume that we are interested in analyzing the attractiveness of three houses that one person is thinking of buying one of them. Now, let us analyze this attractiveness according to our model (IV-Q-NSS), therefore we assume that the three houses present as following universal set $\mathfrak{U} = \{\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{u}_3\}$ and $\mathfrak{Q} = \{\mathfrak{q}_1, \mathfrak{q}_2\}$ be a set constituting two cities under consideration and $\mathcal{E} = \{e_1, e_2, e_3\}$ be a collection of attribute.

$$\hat{P}_{\mathfrak{Q}_{\overline{\mathrm{A}}}} =$$
$$\Bigg\{\Bigg(\mathrm{e}_1, \frac{\langle[0.2,0.8],[0.1,0.7],[0.4,0.8]\rangle}{(\mathfrak{u}_1,\mathfrak{q}_1)}, \frac{\langle[0.1,0.4],[0.5,0.8],[0.7,0.8]\rangle}{(\mathfrak{u}_1,\mathfrak{q}_2)}$$
$$\frac{\langle[0.3,0.6],[0.2,0.7],[0.5,0.8]\rangle}{(\mathfrak{u}_2,\mathfrak{q}_1)}, \frac{\langle[0.4,0.6],[0.2,0.9],[0.5,0.7]\rangle}{(\mathfrak{u}_2,\mathfrak{q}_2)}$$
$$\frac{\langle[0.1,0.5],[0.3,0.7],[0.2,0.8]\rangle}{(\mathfrak{u}_3,\mathfrak{q}_1)}, \frac{\langle[0.4,0.8],[0.4,0.6],[0.2,0.8]\rangle}{(\mathfrak{u}_3,\mathfrak{q}_2)}\Bigg)$$
$$\Bigg(\mathrm{e}_2, \frac{\langle[0.1,0.8],[0.5,0.7],[0.3,0.4]\rangle}{(\mathfrak{u}_1,\mathfrak{q}_1)}, \frac{\langle[0.1,0.8],[0.4,0.7],[0.2,0.6]\rangle}{(\mathfrak{u}_1,\mathfrak{q}_2)}$$
$$\frac{\langle[0.5,0.8],[0.4,0.9],[0.2,0.7]\rangle}{(\mathfrak{u}_2,\mathfrak{q}_1)}, \frac{\langle[0.1,0.2],[0.2,0.5],[0.4,0.7]\rangle}{(\mathfrak{u}_2,\mathfrak{q}_2)}$$
$$\frac{\langle[0.1,0.4],[0.2,0.5],[0.3,0.7]\rangle}{(\mathfrak{u}_3,\mathfrak{q}_1)}, \frac{\langle[0.1,0.6],[0.4,0.5],[0.5,0.7]\rangle}{(\mathfrak{u}_3,\mathfrak{q}_2)}\Bigg)$$
$$\Bigg(\mathrm{e}_3, \frac{\langle[0.7,0.9],[0.2,0.8],[0.3,0.6]\rangle}{(\mathfrak{u}_1,\mathfrak{q}_1)}, \frac{\langle[0.4,0.7],[0.2,0.5],[0.1,0.7]\rangle}{(\mathfrak{u}_1,\mathfrak{q}_2)}$$
$$\frac{\langle[0.1,0.8],[0.1,0.4],[0.3,0.6]\rangle}{(\mathfrak{u}_2,\mathfrak{q}_1)}, \frac{\langle[0.5,0.6],[0.3,0.6],[0.2,0.7]\rangle}{(\mathfrak{u}_2,\mathfrak{q}_2)}$$
$$\frac{\langle[0.4,0.6],[0.2,0.7],[0.3,0.6]\rangle}{(\mathfrak{u}_3,\mathfrak{q}_1)}, \frac{\langle[0.4,0.8],[0.8,0.9],[0.3,0.7]\rangle}{(\mathfrak{u}_3,\mathfrak{q}_2)}\Bigg)\Bigg\}$$

4. Distance and similarity measures for IV-Q-NSSs

In this section, we give out new types of distance and similarity measures on interval-valued Q-neutrosophic soft environments, along with some numerical examples to clarify how these measures work in IV-Q-NSS environments.

Definition 4.1: Let $(\hat{P}_{\mathfrak{Q}}, \bar{A}) = \hat{P}_{\mathfrak{Q}_{\bar{A}}} = \{e \in \bar{A}, \hat{P}_{\mathfrak{Q}_A}^{t}(u,q)(e), \hat{P}_{\mathfrak{Q}_A}^{i}(u,q)(e), \hat{P}_{\mathfrak{Q}_A}^{f}(u,q)(e) > (u,e) \in U \times \mathfrak{Q}\}$ and $(\hat{P}_{\mathfrak{Q}}, \bar{B}) = \hat{P}_{\mathfrak{Q}_{\bar{B}}} = \{e \in \bar{B} < \hat{P}_{\mathfrak{Q}_B}^{t}(u,q)(e), \hat{P}_{\mathfrak{Q}_B}^{i}(u,q)(e), \hat{P}_{\mathfrak{Q}_B}^{f}(u,q)(e) > (u,e) \in U \times \mathfrak{Q}\}$ be two IV-Q-NSSs on the cross product $U \times \mathfrak{Q}$ Then:

i) The Hamming distance

$$D_{IV-QNSS}^{H}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}_{\mathfrak{Q}_{\bar{B}}}\right) = \left|\frac{1}{mn}\sum_{j=1}^{n}\sum_{i=1}^{m} \triangle_{ij}\, \hat{P}_{\mathfrak{Q}}^{t}(u,q)(e) + \triangle_{ij}\, \hat{P}_{\mathfrak{Q}}^{i}(u,q)(e) + \triangle_{ij}\, \hat{P}_{\mathfrak{Q}}^{f}(u,q)(e)\right|$$

Where

$\triangle_{ij}\, \hat{P}^{t}(u,q)(e) = \hat{P}_{\mathfrak{Q}_A}^{t}(u,q)(e) - \hat{P}_{\mathfrak{Q}_B}^{t}(u,q)(e) =$
$\left(\hat{P}_{\mathfrak{Q}_A}^{t,l}(u,q)(e) - \hat{P}_{\mathfrak{Q}_B}^{t,l}(u,q)(e)\right) - \left(\hat{P}_{\mathfrak{Q}_A}^{t,u}(u,q)(e) - \hat{P}_{\mathfrak{Q}_B}^{t,u}(u,q)(e)\right),$
$\triangle_{ij}\, \hat{P}^{i}(u,q)(e) = \hat{P}_{\mathfrak{Q}_A}^{i}(u,q)(e) - \hat{P}_{\mathfrak{Q}_B}^{i}(u,q)(e) =$
$\left(\hat{P}_{\mathfrak{Q}_A}^{i,l}(u,q)(e) - \hat{P}_{\mathfrak{Q}_B}^{i,l}(u,q)(e)\right) - \left(\hat{P}_{\mathfrak{Q}_A}^{i,u}(u,q)(e) - \hat{P}_{\mathfrak{Q}_B}^{i,u}(u,q)(e)\right),$
$\triangle_{ij}\, \hat{P}^{f}(u,q)(e) = \hat{P}_{\mathfrak{Q}_A}^{f}(u,q)(e) - \hat{P}_{\mathfrak{Q}_B}^{f}(u,q)(e) =$
$\left(\hat{P}_{\mathfrak{Q}_A}^{f,l}(u,q)(e) - \hat{P}_{\mathfrak{Q}_B}^{f,l}(u,q)(e)\right) - \left(\hat{P}_{\mathfrak{Q}_A}^{f,u}(u,q)(e) - \hat{P}_{\mathfrak{Q}_B}^{f,u}(u,q)(e)\right).$

ii) The normalized Hamming distance

$$D^{NHD}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}_{\mathfrak{Q}_B}^{t}\right) = \frac{D^{HD}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}_{\mathfrak{Q}_B}^{t}\right)}{mn}$$

iii) The Euclidean distance

$$D_{IV-QNSS}^{ED}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}_{\mathfrak{Q}_{\bar{B}}}\right) = \left|\left(\frac{1}{mn}\sum_{j=1}^{n}\sum_{i=1}^{m}\left(\triangle_{ij}\, \hat{P}_{\mathfrak{Q}}^{t}(u,q)(e)\right)^{2} + \left(\triangle_{ij}\, \hat{P}_{\mathfrak{Q}}^{i}(u,q)(e)\right)^{2} + \left(\triangle_{ij}\, \hat{P}_{\mathfrak{Q}}^{f}(u,q)(e)\right)^{2}\right)^{\frac{1}{2}}\right|$$

Where

$\triangle_{ij}\, \hat{P}^{t}(u,q)(e) = \hat{P}_{\mathfrak{Q}_A}^{t}(u,q)(e) - \hat{P}_{\mathfrak{Q}_B}^{t}(u,q)(e) =$
$\left(\hat{P}_{\mathfrak{Q}_A}^{t,l}(u,q)(e) - \hat{P}_{\mathfrak{Q}_B}^{t,l}(u,q)(e)\right) - \left(\hat{P}_{\mathfrak{Q}_A}^{t,u}(u,q)(e) - \hat{P}_{\mathfrak{Q}_B}^{t,u}(u,q)(e)\right),$
$\triangle_{ij}\, \hat{P}^{i}(u,q)(e) = \hat{P}_{\mathfrak{Q}_A}^{i}(u,q)(e) - \hat{P}_{\mathfrak{Q}_B}^{i}(u,q)(e) =$
$\left(\hat{P}_{\mathfrak{Q}_A}^{i,l}(u,q)(e) - \hat{P}_{\mathfrak{Q}_B}^{i,l}(u,q)(e)\right) - \left(\hat{P}_{\mathfrak{Q}_A}^{i,u}(u,q)(e) - \hat{P}_{\mathfrak{Q}_B}^{i,u}(u,q)(e)\right),$
$\triangle_{ij}\, \hat{P}^{f}(u,q)(e) = \hat{P}_{\mathfrak{Q}_A}^{f}(u,q)(e) - \hat{P}_{\mathfrak{Q}_B}^{f}(u,q)(e) =$
$\left(\hat{P}_{\mathfrak{Q}_A}^{f,l}(u,q)(e) - \hat{P}_{\mathfrak{Q}_B}^{f,l}(u,q)(e)\right) - \left(\hat{P}_{\mathfrak{Q}_A}^{f,u}(u,q)(e) - \hat{P}_{\mathfrak{Q}_B}^{f,u}(u,q)(e)\right).$

iv) The normalized Euclidean distance

$$D_{IV-QNSS}^{NED}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}_{\mathfrak{Q}_B}^{t}\right) = \frac{D^{ED}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}_{\mathfrak{Q}_B}^{t}\right)}{\sqrt{mn}}$$

v) The Hausdorff distance

$$D_{IV-QNSS}^{HD}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}_{\mathfrak{Q}_{\bar{B}}}\right) = \frac{1}{mn}\sum_{j=1}^{n}\sum_{i=1}^{m} max\left(\triangle_{ij}\, \hat{P}_{\mathfrak{Q}}^{t}(u,q)(e), \triangle_{ij}\, \hat{P}_{\mathfrak{Q}}^{i}(u,q)(e), \triangle_{ij}\, \hat{P}_{\mathfrak{Q}}^{f}(u,q)(e)\right)$$

Where

$\left|\triangle_{ij}\, \hat{P}^{t}(u,q)(e)\right| = \left|\hat{P}_{\mathfrak{Q}_A}^{t}(u,q)(e) - \hat{P}_{\mathfrak{Q}_B}^{t}(u,q)(e)\right| =$
$\left|\left(\hat{P}_{\mathfrak{Q}_A}^{t,l}(u,q)(e) - \hat{P}_{\mathfrak{Q}_B}^{t,l}(u,q)(e)\right) - \left(\hat{P}_{\mathfrak{Q}_A}^{t,u}(u,q)(e) - \hat{P}_{\mathfrak{Q}_B}^{t,u}(u,q)(e)\right)\right|,$
$\left|\triangle_{ij}\, \hat{P}^{i}(u,q)(e)\right| = \left|\hat{P}_{\mathfrak{Q}_A}^{i}(u,q)(e) - \hat{P}_{\mathfrak{Q}_B}^{i}(u,q)(e)\right| =$
$\left|\left(\hat{P}_{\mathfrak{Q}_A}^{i,l}(u,q)(e) - \hat{P}_{\mathfrak{Q}_B}^{i,l}(u,q)(e)\right) - \left(\hat{P}_{\mathfrak{Q}_A}^{i,u}(u,q)(e) - \hat{P}_{\mathfrak{Q}_B}^{i,u}(u,q)(e)\right)\right|,$
$\left|\triangle_{ij}\, \hat{P}^{f}(u,q)(e)\right| = \left|\hat{P}_{\mathfrak{Q}_A}^{f}(u,q)(e) - \hat{P}_{\mathfrak{Q}_B}^{f}(u,q)(e)\right| =$

$\left|\left(\hat{P}^{f,l}_{\mathfrak{Q}_A}(u,q)(e) - \hat{P}^{f,l}_{\mathfrak{Q}_B}(u,q)(e)\right) - \left(\hat{P}^{f,u}_{\mathfrak{Q}_A}(u,q)(e) - \hat{P}^{f,u}_{\mathfrak{Q}_B}(u,q)(e)\right)\right|.$

4.1 Distance-based similarity measure (SM) of IV-Q-NSSs

Definition 4.1.1 A real ralue function $\tilde{S}: IV - Q - NSS \times IV - Q - NSS \rightarrow [0,1]^3$ is called a similarity measure (SM) between tow $IV - Q - NSSs$ if then following points are fullified $\forall \hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}^t_{\mathfrak{Q}_B}, \hat{P}_{\mathfrak{Q}_{\bar{C}}} \in IV - QNSSs$, then

1) $0 \leq \tilde{S}(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}^t_{\mathfrak{Q}_B}) \leq 1$
2) $\tilde{S}(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}^t_{\mathfrak{Q}_B}) = 1 \;\; IF \;\; \hat{P}_{\mathfrak{Q}_{\bar{A}}} = \hat{P}^t_{\mathfrak{Q}_B}$
3) $\tilde{S}(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}^t_{\mathfrak{Q}_B}) = \tilde{S}(\hat{P}^t_{\mathfrak{Q}_B}, \hat{P}_{\mathfrak{Q}_{\bar{A}}})$
4) $IF \hat{P}_{\mathfrak{Q}_{\bar{A}}} \subseteq \hat{P}^t_{\mathfrak{Q}_B} \subseteq \hat{P}_{\mathfrak{Q}_{\bar{C}}} \; then \; \tilde{S}(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}^t_{\mathfrak{Q}_B}) \leq \min\{\tilde{S}(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}^t_{\mathfrak{Q}_B}), \tilde{S}(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}^t_{\mathfrak{Q}_C})\}$

1) $\tilde{S}^{H}_{IV-Q-NSS}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}_{\mathfrak{Q}_{\bar{B}}}\right) = \frac{1}{1 + D^{H}_{IV-QNSS}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}_{\mathfrak{Q}_{\bar{B}}}\right)}$
2) $\tilde{S}^{NH}_{IV-Q-NSS}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}_{\mathfrak{Q}_{\bar{B}}}\right) = \frac{1}{1 + D^{NH}_{IV-QNSS}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}_{\mathfrak{Q}_{\bar{B}}}\right)}$
3) $\tilde{S}^{ED}_{IV-Q-NSS}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}_{\mathfrak{Q}_{\bar{B}}}\right) = \frac{1}{1 + D^{ED}_{IV-QNSS}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}_{\mathfrak{Q}_{\bar{B}}}\right)}$
4) $\tilde{S}^{NED}_{IV-Q-NSS}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}_{\mathfrak{Q}_{\bar{B}}}\right) = \frac{1}{1 + D^{NED}_{IV-QNSS}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}_{\mathfrak{Q}_{\bar{B}}}\right)}$
5) $\tilde{S}^{HD}_{IV-Q-NSS}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}_{\mathfrak{Q}_{\bar{B}}}\right) = \frac{1}{1 + D^{HD}_{IV-QNSS}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}_{\mathfrak{Q}_{\bar{B}}}\right)}$

Proprdition4.1.2 For two $IV - ` - NSSs \; \hat{P}_{\mathfrak{Q}_A} and \; \hat{P}_{\mathfrak{Q}_B}$ in universal discourse $\mathcal{U} = \{u_1, u_2, u_3, \ldots, u_n\}$ then $\tilde{S}(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}^t_{\mathfrak{Q}_B})$. Should full fied the following pointes

1) $0 \leq \tilde{S}(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}^t_{\mathfrak{Q}_B}) \leq 1$
2) $\tilde{S}(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}^t_{\mathfrak{Q}_B}) = 1 \;\; IF \;\; \hat{P}_{\mathfrak{Q}_{\bar{A}}} = \hat{P}^t_{\mathfrak{Q}_B}$
3) $\tilde{S}(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}^t_{\mathfrak{Q}_B}) = \tilde{S}(\hat{P}^t_{\mathfrak{Q}_B}, \hat{P}_{\mathfrak{Q}_{\bar{A}}})$

Proof: It is clear from the definitions above

Example 4.1.3 Assume that $\hat{P}_{\mathfrak{Q}_{\bar{A}}} = \langle[0.2,0.8], [0.1,0.7], [0.4,0.8]\rangle$ and $\hat{P}_{\mathfrak{Q}_{\bar{B}}} = \langle[0.1,0.5], [0.3,0.4], [0.2,0.6]\rangle$ be two $IV - Q - NSSs$ on the cross product $U \times \mathfrak{Q}$ mention in Example 3.2 above Then:

i. **The Hamming distance**

$D^{H}_{IV-QNSS}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}_{\mathfrak{Q}_{\bar{B}}}\right) = |(0.2 - 0.1) - (0.8 - 0.5) + (0.1 - 0.3) - (0.7 - 0.4) +$
$(0.4 - 0.2) - (0.8 - 0.6)| = 0.4$

ii. **The normalized Hamming distance**

$D^{NHD}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}^t_{\mathfrak{Q}_B}\right) = \frac{0.4}{9} = 0.04$

iii. **The Euclidean distance**

$D^{ED}_{IV-QNSS}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}_{\mathfrak{Q}_{\bar{B}}}\right) = \sqrt{0.29} = 0.538$

iv. **The normalized Euclidean distance**

$D^{NED}_{IV-QNSS}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}^t_{\mathfrak{Q}_B}\right) = \frac{0.538}{9} = 0.0597$

v. **The Hausdorff distance**

$D_{IV-QNSS}^{HD}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}},\hat{P}_{\mathfrak{Q}_{\bar{B}}}\right)=\frac{1}{9}\Big(max\big((0.2-0.1)-(0.8-0.5),(0.1-0.3)-(0.7-0.4),$
$(0.4-0.2)-(0.8-0.6)\big)\Big)$=0.022

Then, based on output of the above distance values, we can calculate the percentage of similarity between the two components $\hat{P}_{\mathfrak{Q}_{\bar{A}}},\hat{P}_{\mathfrak{Q}_{\bar{B}}}\in$ IV-QNSS as the following:

$\tilde{S}_{IV-Q-NSS}^{H}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}},\hat{P}_{\mathfrak{Q}_{\bar{B}}}\right)=\frac{1}{1+0.4}=0.714,\ \tilde{S}_{IV-Q-NSS}^{NH}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}},\hat{P}_{\mathfrak{Q}_{\bar{B}}}\right)=\frac{1}{1+0.04}=0.961$
$\tilde{S}_{IV-Q-NSS}^{ED}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}},\hat{P}_{\mathfrak{Q}_{\bar{B}}}\right)=\frac{1}{1+0.538}=0.650,\tilde{S}_{IV-Q-NSS}^{NED}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}},\hat{P}_{\mathfrak{Q}_{\bar{B}}}\right)=\frac{1}{1+0.059}=0.944$

Definition 4.1.4 Let $(\hat{P}_{\mathfrak{Q}},\bar{A})=\hat{P}_{\mathfrak{Q}_{\bar{A}}}=\left\{e\in\bar{A},\hat{P}_{\mathfrak{Q}_A}^{t}(u,q)(e),\hat{P}_{\mathfrak{Q}_A}^{i}(u,q)(e),\hat{P}_{\mathfrak{Q}_A}^{f}(u,q)(e)>(u,e)\in U\times\mathfrak{Q}\right\}$ and $(\hat{P}_{\mathfrak{Q}},\bar{B})=\hat{P}_{\mathfrak{Q}_{\bar{B}}}=\left\{e\in\bar{B}<\hat{P}_{\mathfrak{Q}_B}^{t}(u,q)(e),\hat{P}_{\mathfrak{Q}_B}^{i}(u,q)(e),\hat{P}_{\mathfrak{Q}_B}^{f}(u,q)(e)>(u,e)\in U\times\mathfrak{Q}\right\}$ be two IV-Q-NSSs on the cross product $U\times\mathfrak{Q}$ then the cosin similarity measure for $IV-Q-NSSs$ defined as following:

$$\tilde{S}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}},\hat{P}_{\mathfrak{Q}_B}^{t}\right)=\sum_{j=1}^{n}\sum_{i=1}^{n}\cos\Big[\frac{\pi}{12}\left(\left|\hat{P}_{\mathfrak{Q}_A}^{t}(u,q)(e)-\hat{P}_{\mathfrak{Q}_B}^{t}(u,q)(e)\right|\right)+\left(\left|\hat{P}_{\mathfrak{Q}_A}^{i}(u,q)(e)-\hat{P}_{\mathfrak{Q}_B}^{i}(u,q)(e)\right|\right)+\left(\left|\hat{P}_{\mathfrak{Q}_A}^{f}(u,q)(e)-\hat{P}_{\mathfrak{Q}_B}^{f}(u,q)(e)\right|\right)\Big]$$

$$=\sum_{j=1}^{n}\sum_{i=1}^{n}\cos\ \frac{\pi}{12}\Big(\left|\hat{P}_{\mathfrak{Q}_A}^{t,l}(u,q)(e)-\hat{P}_{\mathfrak{Q}_B}^{t,l}(u,q)(e)\right|+\left|\hat{P}_{\mathfrak{Q}_A}^{i,l}(u,q)(e)-\hat{P}_{\mathfrak{Q}_B}^{i,l}(u,q)(e)\right|+\left|\hat{P}_{\mathfrak{Q}_A}^{f,l}(u,q)(e)-\hat{P}_{\mathfrak{Q}_B}^{f,l}(u,q)(e)\right|+\left|\hat{P}_{\mathfrak{Q}_A}^{t,l}(u,q)(e)-\hat{P}_{\mathfrak{Q}_B}^{t,l}(u,q)(e)\right|+\left|\hat{P}_{\mathfrak{Q}_A}^{i,l}(u,q)(e)-\hat{P}_{\mathfrak{Q}_B}^{i,l}(u,q)(e)\right|+\left|\hat{P}_{\mathfrak{Q}_A}^{f,l}(u,q)(e)-\hat{P}_{\mathfrak{Q}_B}^{f,l}(u,q)(e)\right|\Big)$$

Example 4.1.5 Assume that $\hat{P}_{\mathfrak{Q}_{\bar{A}}}=\langle[0.2,0.8],[0.1,0.7],[0.4,0.8]\rangle$ and $\hat{P}_{\mathfrak{Q}_{\bar{B}}}=\langle[0.1,0.5],[0.3,0.4],[0.2,0.6]\rangle$ be two $IV-Q-NSSs$ on the cross product $U\times\mathfrak{Q}$ mention in Example 1 above Then:

$$\tilde{S}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}},\hat{P}_{\mathfrak{Q}_B}^{t}\right)=\sum_{j=1}^{n}\sum_{i=1}^{n}\cos\ \frac{\pi}{12}(|0.2-0.1|+|0.1-0.3|+|0.4-0.2|+|0.8-0.5|+|0.7-0.4|+|0.8-0.6|)$$
$=0.869$

4.2. A practical application of the tools presented in this work

In this section, we will design a new algorithm that depends on the tools presented in the work. After that, we use this algorithm to solve one of the decision-making problems related to testing the efficiency of electrical appliances. Below is a structural diagram showing the steps of the algorithm:

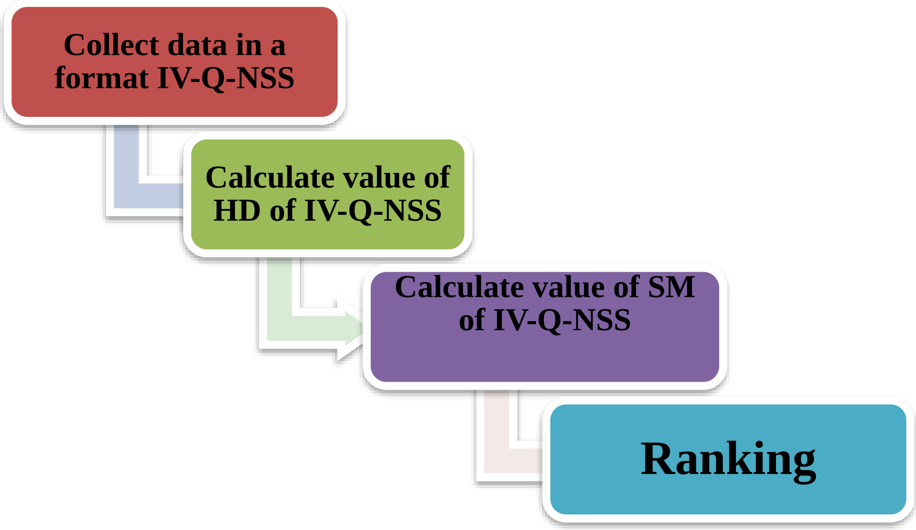

Figure 2. Represents the proposed algorithm

Cause study:
One of the companies selling electrical appliances, which imports all these devices from different countries, decided to carry out an evaluation process for these devices based on some international standards. In any case, to carry out this process requires the assistance of experts who have the ability to analyze people's opinions about this product. We must assume that the number of devices to be evaluated is 4, which can be represented by $\mathfrak{U} = \{\mathfrak{u}_1, \mathfrak{u}_2, \mathfrak{u}_3\}$ and the number of people participating in the questionnaire is 2, which can be represented by $\mathfrak{Q} = \{\mathfrak{q}_1, \mathfrak{q}_2\}$. As for the criteria that were adopted, they are as follows $\mathcal{E} = \{e_1, e_2, e_3\}$ where e_1 =Device age, e_2 =Manufacturing country and e_3 =Device cost. Accordingly, a questionnaire was conducted on two users who gave their opinions on this issue in complete transparency. On this basis, the user analyzed these opinions and converted them into values that could be represented by our proposed model (IV-Q-NSS) as following:

$$\hat{P}_{\mathfrak{Q}_{\bar{B}}} = \left\{ \left(e_1, \frac{\langle[0.2,0.8],[0.1,0.7],[0.4,0.8]\rangle}{(\mathfrak{u}_1,\mathfrak{q}_1)}, \frac{\langle[0.1,0.4],[0.5,0.8],[0.7,0.8]\rangle}{(\mathfrak{u}_1,\mathfrak{q}_2)}, \frac{\langle[0.3,0.6],[0.2,0.7],[0.5,0.8]\rangle}{(\mathfrak{u}_2,\mathfrak{q}_1)}, \frac{\langle[0.4,0.6],[0.2,0.9],[0.5,0.7]\rangle}{(\mathfrak{u}_2,\mathfrak{q}_2)}, \frac{\langle[0.1,0.5],[0.3,0.7],[0.2,0.8]\rangle}{(\mathfrak{u}_3,\mathfrak{q}_1)}, \frac{\langle[0.4,0.8],[0.4,0.6],[0.2,0.8]\rangle}{(\mathfrak{u}_3,\mathfrak{q}_2)} \right) \left(e_2, \frac{\langle[0.1,0.8],[0.5,0.7],[0.3,0.4]\rangle}{(\mathfrak{u}_1,\mathfrak{q}_1)}, \frac{\langle[0.1,0.8],[0.4,0.7],[0.2,0.6]\rangle}{(\mathfrak{u}_1,\mathfrak{q}_2)}, \frac{\langle[0.5,0.8],[0.4,0.9],[0.2,0.7]\rangle}{(\mathfrak{u}_2,\mathfrak{q}_1)}, \frac{\langle[0.1,0.2],[0.2,0.5],[0.4,0.7]\rangle}{(\mathfrak{u}_2,\mathfrak{q}_2)}, \frac{\langle[0.1,0.4],[0.2,0.5],[0.3,0.7]\rangle}{(\mathfrak{u}_3,\mathfrak{q}_1)}, \frac{\langle[0.1,0.6],[0.4,0.5],[0.5,0.7]\rangle}{(\mathfrak{u}_3,\mathfrak{q}_2)} \right) \left(e_3, \frac{\langle[0.7,0.9],[0.2,0.8],[0.3,0.6]\rangle}{(\mathfrak{u}_1,\mathfrak{q}_1)}, \frac{\langle[0.4,0.7],[0.2,0.5],[0.1,0.7]\rangle}{(\mathfrak{u}_1,\mathfrak{q}_2)}, \frac{\langle[0.1,0.8],[0.1,0.4],[0.3,0.6]\rangle}{(\mathfrak{u}_2,\mathfrak{q}_1)}, \frac{\langle[0.5,0.6],[0.3,0.6],[0.2,0.7]\rangle}{(\mathfrak{u}_2,\mathfrak{q}_2)}, \frac{\langle[0.4,0.6],[0.2,0.7],[0.3,0.6]\rangle}{(\mathfrak{u}_3,\mathfrak{q}_1)}, \frac{\langle[0.4,0.8],[0.8,0.9],[0.3,0.7]\rangle}{(\mathfrak{u}_3,\mathfrak{q}_2)} \right) \right\}$$

Solved case study: Now we apply the above algorithm by finding the HD and SM between the values in each term and the ideal value$\hat{P}_{\mathfrak{Q}_{\bar{A}}} = [1,1]$ as mention in Table 1.

Table 1. Table captions should be placed above the tables.

Opinions	**HD**	**SM**	**Ranking**
$(\mathfrak{u}_1, \mathfrak{q}_1)$	**1.119**	**0.471**	**1**
$(\mathfrak{u}_1, \mathfrak{q}_2)$	**1.320**	**0.431**	**2**
$(\mathfrak{u}_2, \mathfrak{q}_1)$	**1.457**	**0.407**	**2**
$(\mathfrak{u}_2, \mathfrak{q}_2)$	**1.094**	**0.477**	**1**
$(\mathfrak{u}_3, \mathfrak{q}_1)$	**1.722**	**0.367**	**3**
$(\mathfrak{u}_3, \mathfrak{q}_2)$	**1.582**	**0.387**	**3**

Compute HD and SHD showing in Table 1 using following formulas

$$D^{H}_{IV-QNSS}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}_{\mathfrak{Q}_{\bar{B}}}\right) = \frac{1}{9}|(1-0.2)-(1-0.8)+(1-0.1)-(1-0.7)$$
$$+(1-0.4)+(1-0.8)-(1-0.1)-(1-0.8)+(1-0.5)-(1-0.7)$$
$$-(1-0.3)+(1-0.4)-(1-0.7)-(1-0.9)+(1-0.2)-(1-0.8)$$
$$-(1-0.3)+(1-0.6)+(1-0.4)-(1-0.6)+(1-0.2)-(1-0.7)$$
$$+(1-0.3)\ -(1-0.8)| = 1.119$$
$$\tilde{S}^{H}_{IV-Q-NSS}\left(\hat{P}_{\mathfrak{Q}_{\bar{A}}}, \hat{P}_{\mathfrak{Q}_{\bar{B}}}\right) = \frac{1}{1+1.119} = 0.471$$

From Table 1, we note that the device $\mathfrak{u}_2$ is of the best quality according to the opinions of the residents $\mathfrak{q}_1, \mathfrak{q}_2$.

5. Conclusion

In this work, we illustrated a new mathematical model named interval valued-Q-neutrosophic soft sets (IV-Q-NSSs) as a hyperd fuzzy extension of some previous models. After that, we discovered the distance and similarity measures on IV-Q-NSSs. We proposed several types of these measures and illustrated them with several numerical examples. Based on these measures, we created an innovative algorithm based on these procedures to solve one of the problems of daily life.

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