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# An Intuitionistic Fuzzy Parameterized Decision-making Approach For Evaluating Site Suitability For Educational Institutions Using Hypersoft Settings

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**ABSTRACT**: Intuitionistic fuzzy parameterization aims to evaluate the uncertain and ambiguous nature of parameters and sub-parameters during the decision-making process. In this parameterization, an intuitionistic fuzzy grade is assigned to parameters and sub-parameters to address ambiguity. In cases of hypersoft settings, this grade is attached to multi-argument tuples containing sub-parametric values. The integration of intuitionistic fuzzy parameterization with hypersoft settings offers a flexible framework for modeling uncertainties in decision-making scenarios. In this study, a new theoretical framework called the intuitionistic fuzzy parameterized hypersoft set (IFpHSS) is developed, which replaces existing concepts based on parameterization of intuitionistic fuzzy soft sets. The concepts of empty, universal, subset, and equal IFpHSS are introduced and clarified with examples. Aggregation operations such as complement, union, intersection, AND, and OR for IFpHSS are also explained. Using the idea of an intuitionistic fuzzy decision set, cardinality set, and reduced fuzzy set within the IFpHSS framework, a robust decision-support system is established, along with an intelligent algorithm. This algorithm is applied to evaluate different sites for the establishment of an educational institution, demonstrating its effectiveness for solving problems involving vagueness and uncertainties.

## 1. INTRODUCTION

The hypersoft set [1] is a significant advancement in the field of mathematical modeling of uncertainty, particularly building upon and extending soft set [2] theory. In traditional soft set theory, parameters are used to represent uncertain or vague information. However, this model treats each parameter as a singular entity, limiting its ability to handle complex and hierarchical information structures. The hypersoft set addresses this limitation by introducing a multi-argument domain, where each parameter can be associated with multiple sub-parametric values or tuples. This allows a finer and more detailed approximation of uncertain information. For instance, instead of evaluating a decision based solely on a parameter like "cost," a hypersoft set allows the decision-maker to consider sub-components such as "initial investment," "maintenance cost," and "operating expenses" as a composite tuple under the main parameter. This refined granularity greatly enhances the modeling power of the set, enabling decision-makers to capture more nuanced aspects of a problem. By incorporating sub-parametric structures, the hypersoft set provides a more flexible and realistic framework to deal with ambiguity, incomplete information, and overlapping domains. Moreover, in multi-attribute decision-making (MADM) scenarios, the hypersoft set helps evaluate options with higher accuracy and discrimination. It does so by allowing decisions to be influenced by a set of interconnected sub-attributes, leading to a better representation of human-like reasoning and judgment under uncertainty.

The intuitionistic fuzzy hypersoft set (IFHSS) [3] plays a vital role in solving MADM problems by effectively handling ambiguity, hesitation, and imprecise information. Unlike traditional models [4–7], IFHSS integrates the strength of hypersoft sets—through multi-argument sub-parametric structures—with intuitionistic fuzzy logic, which captures both membership and non-membership degrees along with hesitation. This enriched structure enables decision-makers to model complex systems more accurately, evaluate alternatives with finer granularity, and make more balanced and in-

formed choices in environments characterized by uncertainty and conflicting attributes.1.00,0.00,0.00 Recently, Ihsan et al. [8] discussed the evaluation of suitable candidates for a job by integrating modified TOPSIS and the correlation coefficient of IFHSS with multi-decisive opinion settings. Arshad et al. [9] introduced a novel hybrid framework of IFHSS with complex and interval-valued settings. They applied the proposed idea to the evaluation of cooling systems. Sajid et al. [10] discussed the evaluation of solar panels using aggregation of IFHSS with cubic settings. Zulqarnain & Siddique [11] and Naveed & Ali [12] explored MADM problems using a weighted average interaction aggregation operator and similarity measures of IFHSS with interval-valued settings, respectively. Rani & Periyasamy [13] developed the theory of matrices for IFHSS and employed it in MADM. Ali et al. [14], Jafar et al. [15], Naveed & Ali [16], Saeed et al. [17], and Jafar & Saeed [18] investigated geometric aggregation operators of interval-valued IFHSS, cosine and cotangent similarity measures of IFHSS, similarity measures for interval-valued IFHSS, hamming and hausdorff distance metrics for cubic IFHSS, and aggregation operations of IFHSS, respectively.
Intuitionistic fuzzy parameterization is a refined approach used in uncertain decision-making environments, where the evaluation of alternatives involves parameters and their corresponding sub-parameters that are often vague, imprecise, or incomplete. In this parameterization, each parameter and sub-parameter in a decision-making problem is assigned an intuitionistic fuzzy grade—a pair of values representing how much it supports (membership) or contradicts (non-membership) a certain alternative, along with the hesitation degree derived from the incomplete knowledge. This structure allows for a more comprehensive representation of uncertainty, especially in complex systems where attributes cannot be fully classified as true or false. By modeling ambiguity through intuitionistic fuzzy grades, this approach enhances the reliability and depth of the evaluation process. It enables decision-makers to capture not only the positive and negative aspects of an attribute but also the level of indecision, leading to more informed, transparent, and rational decisions, particularly in scenarios involving conflicting or partial expert opinions. The integration of intuitionistic fuzzy parameterization with hypersoft settings represents a significant advancement in modeling complex and uncertain decision-making environments. Traditional soft and fuzzy set-based models [19–25] often fall short when parameters involve multi-level sub-attributes and the available information is imprecise, contradictory, or hesitant. Hypersoft sets extend soft sets by allowing parameters to have multiple sub-parameter values, creating a multi-argument domain that can capture more detailed and realistic relationships. Inspiring from the literature on fuzzy set-like parameterization [26–29], a new theoretical structure known as intuitionistic fuzzy parameterized hypersoft set (IFpHSS) is developed.
1.00,0.00,0.00Unlike traditional intuitionistic fuzzy soft sets (IFSS), which assign membership and non-membership degrees only to single-level parameters, the proposed framework, IFpHSS, introduces a hierarchical structure that allows these intuitionistic fuzzy values to be associated not only with primary parameters but also with their corresponding sub-parameters. This multi-level representation enables the modeling of uncertainty, hesitation, and partial knowledge in a more detailed and structured manner, which is particularly important when dealing with real-world decision-making scenarios that involve complex and nested criteria. The IFpHSS model significantly extends the expressive power of earlier models such as IFSS and IFHSS by capturing the nuanced relationships between a main attribute and its finer subcomponents. In many decision-making problems, especially in MADM, decisions are not made solely based on broad criteria but also depend on sub-criteria that influence outcomes in subtle but crucial ways. For example, when evaluating job applicants, a main criterion like "experience" may branch into sub-criteria such as "technical experience," "management experience," and "industry-specific experience," each with varying degrees of importance and associated uncertainty. By incorporating intuitionistic fuzzy degrees (membership, and non-membership) at both levels, the IFpHSS framework provides a deeper, more granular understanding of each decision alternative. This leads to more robust modeling of expert opinions or stakeholder judgments, accounting not only for agreement or disagreement but also for hesitation or lack of information which is an essential aspect in ambiguous or imprecise environments. Furthermore, the enhanced adaptability and hierarchical expressiveness of IFpHSS make it well-suited for applications in various MADM contexts, such as healthcare diagnosis, project selection, environmental assessment, and risk evaluation. Its structure supports nuanced preference modeling, flexible

parameter interaction, and more comprehensive evaluations, thus helping decision-makers arrive at conclusions that are not only data-driven but also context-sensitive and transparent. In summary, the IFpHSS framework transforms the classical decision-making landscape by embedding deeper layers of intuitionistic fuzzy logic into complex parameter hierarchies, leading to more refined, justified, and interpretable decision outcomes.

### 1.1. SALIENT CONTRIBUTIONS

The key contributions of the paper are stated below:

1. A novel theoretical model called the intuitionistic fuzzy parameterized hypersoft set (IFpHSS) is introduced, which enhances existing intuitionistic fuzzy soft set approaches by incorporating hypersoft structures for handling complex decision-making scenarios involving sub-parameters.
2. Essential concepts such as empty, universal, subset, and equality for IFpHSS are formally defined and illustrated with examples. Furthermore, core aggregation operations (complement, union, intersection, AND, OR) are extended and explained within the context of IFpHSS.
3. A comprehensive decision-making framework is developed using new constructs—intuitionistic fuzzy decision set, cardinality set, and reduced fuzzy set—within the IFpHSS environment, enabling structured analysis under ambiguity and uncertainty.
4. An intelligent algorithm based on the IFpHSS framework is proposed and successfully applied to a real-world case study involving site selection for an educational institution, demonstrating the practical utility and effectiveness of the approach in decision-making under uncertainty.

### 1.2. ORGANIZATION OF PAPER

The remaining paper is organized as follows: Section 2 reviews essential definitions to help readers understand the main idea. Section 3 introduces the notions of IFpHSS, its properties, and aggregation operations, along with examples. Section 4 presents a support system with a proposed robust algorithm for finding a suitable location for an educational institution. Section 5 concludes the paper with a summary and future directions.

## 2. ELEMENTARY KNOWLEDGE

This section revisits some fundamental definitions and key terminologies to ensure clarity and facilitate a better understanding of the main results presented later in the study. The symbols like $\widetilde{\mathscr{L}}$, and $\mathbb{P}^{(\widetilde{\mathscr{L}})}$ will remain consistent in the whole paper and will represent set of alternatives, and power set of set of alternatives, respectively.

**Definition 1.** *[5]*
*For every element $\hat{\ell}$ of set of alternatives $\widetilde{\mathscr{L}}$, an intuitionistic fuzzy set $\hat{\mathscr{Y}}$ is stated as $\hat{\mathscr{Y}} = \{(\hat{\ell}, \langle \hat{\Psi}_{\hat{\mathscr{Y}}}(\hat{\ell}), \hat{\chi}_{\hat{\mathscr{Y}}}(\hat{\ell})\rangle)|\hat{\ell} \in \widetilde{\mathscr{L}}\}$, where $\hat{\Psi}_{\hat{\mathscr{Y}}}$ and $\hat{\chi}_{\hat{\mathscr{Y}}}$ are truth and false membership functions defined by $\hat{\Psi}_{\hat{\mathscr{Y}}} : \widetilde{\mathscr{L}} \to [0,1]$ and $\hat{\chi}_{\hat{\mathscr{Y}}} : \widetilde{\mathscr{L}} \to [0,1]$ respectively. The sum of their values $\hat{\Psi}_{\hat{\mathscr{Y}}}(\hat{\ell})$ and $\hat{\chi}_{\hat{\mathscr{Y}}}(\hat{\ell})$ should lie within [0,1] with hesitancy grade $1 - [\hat{\Psi}_{\hat{\mathscr{Y}}}(\hat{\ell}) + \hat{\chi}_{\hat{\mathscr{Y}}}(\hat{\ell})]$.*
*If $\hat{\mathscr{Y}}_1$ and $\hat{\mathscr{Y}}_2$ are two intuitionistic fuzzy sets then*

1. $\hat{\mathscr{Y}}_1 \cup \hat{\mathscr{Y}}_2 = \{(\hat{\ell}, \langle max\{\hat{\Psi}_{\hat{\mathscr{Y}}_1}(\hat{\ell}), \hat{\Psi}_{\hat{\mathscr{Y}}_2}(\hat{\ell})\}, min\{\hat{\chi}_{\hat{\mathscr{Y}}_1}(\hat{\ell}), \hat{\chi}_{\hat{\mathscr{Y}}_2}(\hat{\ell})\}\rangle)|\hat{\ell} \in \widetilde{\mathscr{L}}\}.$
2. $\hat{\mathscr{Y}}_1 \cap \hat{\mathscr{Y}}_2 = \{(\hat{\ell}, \langle min\{\hat{\Psi}_{\hat{\mathscr{Y}}_1}(\hat{\ell}), \hat{\Psi}_{\hat{\mathscr{Y}}_2}(\hat{\ell})\}, max\{\hat{\chi}_{\hat{\mathscr{Y}}_1}(\hat{\ell}), \hat{\chi}_{\hat{\mathscr{Y}}_2}(\hat{\ell})\}\rangle)|\hat{\ell} \in \widetilde{\mathscr{L}}\}.$
3. $\hat{\mathscr{Y}}^c = \{(\hat{\ell}, \langle \hat{\chi}_{\hat{\mathscr{Y}}}(\hat{\ell}), \hat{\Psi}_{\hat{\mathscr{Y}}}(\hat{\ell})\rangle)|\hat{\ell} \in \widetilde{\mathscr{L}}\}.$

**Definition 2.** *[1]*
*Let $\text{æ}_1, \text{æ}_2, \text{æ}_3, ..., \text{æ}_n$ be different evaluation parameters whose sub-parametric values are respectively, contained in the non-intersecting sets $\widetilde{\mathscr{A}_1}, \widetilde{\mathscr{A}_2}, \widetilde{\mathscr{A}_3}, ..., \widetilde{\mathscr{A}_n}$. The hypersoft set $\widehat{\mathscr{H}}$ is stated as*

$$\widehat{\mathscr{H}} = \left\{ \left(\widehat{\psi}_{\widetilde{\mathscr{H}}}, \widetilde{\mathscr{A}}\right) : \widehat{\psi}_{\widetilde{\mathscr{H}}}(\hat{\alpha}) \subseteq \mathbb{P}^{(\widetilde{\mathscr{L}})} \ \forall \ \hat{\alpha} \in \widetilde{\mathscr{A}} \right\}$$

*where* $\widetilde{\mathscr{A}} = \widetilde{\mathscr{A}_1} \times \widetilde{\mathscr{A}_2} \times \widetilde{\mathscr{A}_3} \times ... \times \widetilde{\mathscr{A}_n}$ *is a multi-argument domain of approximate mapping* $\widehat{\psi}_{\widetilde{\mathscr{H}}} : \widetilde{\mathscr{A}} \rightarrow \mathbb{P}^{(\widetilde{\mathscr{L}})}$.

**Example 1.** *1.00,0.00,0.00Let* $\widetilde{\mathscr{L}} = \{\hat{\ell}_1, \hat{\ell}_2, \hat{\ell}_3, \hat{\ell}_4, \hat{\ell}_5, \hat{\ell}_6\}$ *be the initial set of alternatives and* $\text{æ}_1, \text{æ}_2, \text{æ}_3$ *be three different evaluation parameters. Their sub-parametric values are respectively contained in the non-intersecting sets* $\widetilde{\mathscr{A}_1} = \{\text{æ}_{11}, \text{æ}_{12}\}$, $\widetilde{\mathscr{A}_2} = \{\text{æ}_{21}, \text{æ}_{22}\}$, *and* $\widetilde{\mathscr{A}_3} = \{\text{æ}_{31}\}$. *To obtain multi-argument domain, the set* $\widetilde{\mathscr{A}} = \widetilde{\mathscr{A}_1} \times \widetilde{\mathscr{A}_2} \times \widetilde{\mathscr{A}_3}$ *is calculated that is* $\widetilde{\mathscr{A}} = \{\hat{\alpha}_1 = (\text{æ}_{11}, \text{æ}_{21}, \text{æ}_{31}), \hat{\alpha}_2 = (\text{æ}_{11}, \text{æ}_{22}, \text{æ}_{31}), \hat{\alpha}_3 = (\text{æ}_{12}, \text{æ}_{21}, \text{æ}_{31}), \hat{\alpha}_4 = (\text{æ}_{12}, \text{æ}_{22}, \text{æ}_{31})\}$ *and their approximations are* $\widehat{\psi}_{\widetilde{\mathscr{H}}}(\hat{\alpha}_1) = \{\hat{\ell}_1, \hat{\ell}_3, \hat{\ell}_5\}$, $\widehat{\psi}_{\widetilde{\mathscr{H}}}(\hat{\alpha}_2) = \{\hat{\ell}_2, \hat{\ell}_4, \hat{\ell}_6\}$, $\widehat{\psi}_{\widetilde{\mathscr{H}}}(\hat{\alpha}_3) = \{\hat{\ell}_1, \hat{\ell}_2, \hat{\ell}_6\}$, *and* $\widehat{\psi}_{\widetilde{\mathscr{H}}}(\hat{\alpha}_4) = \{\hat{\ell}_2, \hat{\ell}_4, \hat{\ell}_5\}$. *Thus, based on these approximations, the hypersoft set* $\widetilde{\mathscr{H}}$ *is constructed as*

$\widetilde{\mathscr{H}} = \left\{ \left(\hat{\alpha}_1, \widehat{\psi}_{\widetilde{\mathscr{H}}}(\hat{\alpha}_1)\right), \left(\hat{\alpha}_2, \widehat{\psi}_{\widetilde{\mathscr{H}}}(\hat{\alpha}_2)\right), \left(\hat{\alpha}_3, \widehat{\psi}_{\widetilde{\mathscr{H}}}(\hat{\alpha}_3)\right), \left(\hat{\alpha}_4, \widehat{\psi}_{\widetilde{\mathscr{H}}}(\hat{\alpha}_4)\right) \right\}$ *or*

$\widetilde{\mathscr{H}} = \left\{ \left(\hat{\alpha}_1, \{\hat{\ell}_1, \hat{\ell}_3, \hat{\ell}_5\}\right), \left(\hat{\alpha}_2, \{\hat{\ell}_2, \hat{\ell}_4, \hat{\ell}_6\}\right), \left(\hat{\alpha}_3, \{\hat{\ell}_1, \hat{\ell}_2, \hat{\ell}_6\}\right), \left(\hat{\alpha}_4, \{\hat{\ell}_2, \hat{\ell}_4, \hat{\ell}_5\}\right) \right\}$.

**Definition 3.** *[3]*
*Let* $\widetilde{\Omega}_{IF}$ *be the family of intuitionistic fuzzy sets over* $\widetilde{\mathscr{L}}$ *and* $\text{æ}_1, \text{æ}_2, \text{æ}_3, ..., \text{æ}_n$ *be different evaluation parameters whose sub-parametric values are respectively, contained in the non-intersecting sets* $\widetilde{\mathscr{A}_1}, \widetilde{\mathscr{A}_2}, \widetilde{\mathscr{A}_3}, ..., \widetilde{\mathscr{A}_n}$. *The intuitionistic fuzzy hypersoft set* $\widetilde{\mathscr{I}}$ *is stated as* $\widetilde{\mathscr{I}} = \left\{ \left(\widehat{\psi}_{\widetilde{\mathscr{I}}}, \widetilde{\mathscr{A}}\right) : \widehat{\psi}_{\widetilde{\mathscr{I}}}(\hat{\alpha}) \subseteq \widetilde{\Omega}_{IF} \ \forall \ \hat{\alpha} \in \widetilde{\mathscr{A}} \right\}$ *where* $\widetilde{\mathscr{A}} = \widetilde{\mathscr{A}_1} \times \widetilde{\mathscr{A}_2} \times \widetilde{\mathscr{A}_3} \times ... \times \widetilde{\mathscr{A}_n}$ *is a multi-argument domain of approximate mapping* $\widehat{\psi}_{\widetilde{\mathscr{I}}} : \widetilde{\mathscr{A}} \rightarrow \mathbb{P}^{(\widetilde{\mathscr{L}})}$.

## 3. NOTIONS OF IFPHSS

The purpose of this section is to introduce the concept of IFpHSS. In addition to explaining this concept, the section will discuss some fundamental properties and elements related to IFpHSS. Understanding these fundamentals is essential for grasping how IFpHSS operates.

**Definition 4.** *Let* $\text{æ}_1, \text{æ}_2, \text{æ}_3, ..., \text{æ}_n$ *be different evaluation parameters whose sub-parametric values are respectively, contained in the non-intersecting sets* $\widetilde{\mathscr{A}_1}, \widetilde{\mathscr{A}_2}, \widetilde{\mathscr{A}_3}, ..., \widetilde{\mathscr{A}_n}$. *The IFpHSS* $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}}$ *is stated as* $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}} = \left\{ \left(\widehat{\psi}_{\widetilde{\mathscr{B}}}, \widetilde{\mathscr{B}}\right) : \widehat{\psi}_{\widetilde{\mathscr{B}}}(\hat{\wp}) \subseteq \mathbb{P}^{(\widetilde{\mathscr{L}})} \ \forall \ \hat{\wp} \in \widetilde{\mathscr{B}} \right\}$ *where* $\widetilde{\mathscr{A}} = \widetilde{\mathscr{A}_1} \times \widetilde{\mathscr{A}_2} \times \widetilde{\mathscr{A}_3} \times ... \times \widetilde{\mathscr{A}_n}$ *and* $\widetilde{\mathscr{B}}$ *is an intuitionistic fuzzy set over* $\widetilde{\mathscr{A}}$ *containing elements of the form* $\hat{\wp} = \langle \hat{\Psi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}), \hat{\chi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}) \rangle / \hat{\alpha}$ *for all* $\hat{\alpha} \in \widetilde{\mathscr{A}}$. *The values* $\hat{\Psi}_{\widetilde{\mathscr{B}}}(\hat{\alpha})$ *and* $\hat{\chi}_{\widetilde{\mathscr{B}}}(\hat{\alpha})$ *are respectively truth membership and false membership grades of* $\hat{\alpha} \in \widetilde{\mathscr{A}}$ *such that* $\hat{\Psi}_{\widetilde{\mathscr{A}}}(\hat{\alpha}) + \hat{\chi}_{\widetilde{\mathscr{A}}}(\hat{\alpha}) \in [0, 1]$. *The set* $\widetilde{\mathscr{B}}$ *is an intuitionistic fuzzy valued multi-argument domain of approximate mapping* $\widehat{\psi}_{\widetilde{\mathscr{B}}} : \widetilde{\mathscr{B}} \rightarrow \mathbb{P}^{(\widetilde{\mathscr{L}})}$. *Note that collection of all IFpHSSs is represented by* $\Theta_{IFPHS}(\widetilde{\mathscr{L}})$.

**Example 2.** *Let* $\widetilde{\mathscr{Q}} = \{\text{æ}_1, \text{æ}_2, \text{æ}_3\}$ *and* $\widetilde{\mathscr{L}} = \{\hat{\ell}_1, \hat{\ell}_2, \hat{\ell}_3, \hat{\ell}_4, \hat{\ell}_5, \hat{\ell}_6, \hat{\ell}_7, \hat{\ell}_8\}$ *be a set of parameters and a set of alternatives (objects), respectively. The sub-parametric values of parameters are contained in the non-intersecting sets* $\widetilde{\mathscr{A}_1} = \{\text{æ}_{11}, \text{æ}_{12}\}$, $\widetilde{\mathscr{A}_2} = \{\text{æ}_{21}, \text{æ}_{22}\}$ *and* $\widetilde{\mathscr{A}_3} = \{\text{æ}_{31}\}$. *To find multi-argument domain, the set* $\widetilde{\mathscr{A}} = \widetilde{\mathscr{A}_1} \times \widetilde{\mathscr{A}_2} \times \widetilde{\mathscr{A}_3} = \{\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3, \hat{\alpha}_4\}$ *is determined where* $\hat{\alpha}_1 = (\text{æ}_{11}, \text{æ}_{21}, \text{æ}_{31})$, $\hat{\alpha}_2 = (\text{æ}_{11}, \text{æ}_{22}, \text{æ}_{31})$, $\hat{\alpha}_3 = (\text{æ}_{12}, \text{æ}_{21}, \text{æ}_{31})$, *and* $\hat{\alpha}_4 = (\text{æ}_{12}, \text{æ}_{22}, \text{æ}_{31})$. *According to opinions provided by experts, an intuitionistic fuzzy set* $\widetilde{\mathscr{B}} = \{\hat{\wp}_1, \hat{\wp}_2, \hat{\wp}_3, \hat{\wp}_4\}$ *is constructed over* $\widetilde{\mathscr{A}}$ *where* $\hat{\wp}_1 = \langle 0.451, 0.342 \rangle / \hat{\alpha}_1$, $\langle 0.557, 0.251 \rangle / \hat{\alpha}_2$, $\langle 0.781, 0.131 \rangle / \hat{\alpha}_3$, *and* $\langle 0.635, 0.311 \rangle / \hat{\alpha}_4$. *Now the approximations of these intuitionistic fuzzy valued multi-argument elements are given as*

$$\begin{aligned}
\widehat{\psi}_{\widetilde{\mathscr{B}}}(\hat{\wp}_1) &= \widehat{\psi}_{\widetilde{\mathscr{B}}}(\langle 0.451, 0.342 \rangle / \hat{\alpha}_1) = \{\hat{\ell}_1, \hat{\ell}_3, \hat{\ell}_6\}, \\
\widehat{\psi}_{\widetilde{\mathscr{B}}}(\hat{\wp}_2) &= \widehat{\psi}_{\widetilde{\mathscr{B}}}(\langle 0.557, 0.251 \rangle / \hat{\alpha}_2) = \{\hat{\ell}_2, \hat{\ell}_3, \hat{\ell}_7\}, \\
\widehat{\psi}_{\widetilde{\mathscr{B}}}(\hat{\wp}_3) &= \widehat{\psi}_{\widetilde{\mathscr{B}}}(\langle 0.781, 0.131 \rangle / \hat{\alpha}_3) = \{\hat{\ell}_4, \hat{\ell}_5, \hat{\ell}_8\}, \\
\widehat{\psi}_{\widetilde{\mathscr{B}}}(\hat{\wp}_4) &= \widehat{\psi}_{\widetilde{\mathscr{B}}}(\langle 0.635, 0.311 \rangle / \hat{\alpha}_4) = \{\hat{\ell}_1, \hat{\ell}_7, \hat{\ell}_8\}.
\end{aligned}$$

*Based on these approximations, an IFpHSS* $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}}$ *is constructed as*

$$\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}} = \left\{ \begin{array}{c} \left(\langle 0.451, 0.342 \rangle / \hat{\alpha}_1, \{\hat{\ell}_1, \hat{\ell}_3, \hat{\ell}_6\}\right), \left(\langle 0.557, 0.251 \rangle / \hat{\alpha}_2, \{\hat{\ell}_2, \hat{\ell}_3, \hat{\ell}_7\}\right), \\ \left(\langle 0.781, 0.131 \rangle / \hat{\alpha}_3, \{\hat{\ell}_4, \hat{\ell}_5, \hat{\ell}_8\}\right), \left(\langle 0.635, 0.311 \rangle / \hat{\alpha}_4, \{\hat{\ell}_1, \hat{\ell}_7, \hat{\ell}_8\}\right) \end{array} \right\}.$$

*In this set, $\langle 0.451, 0.342\rangle / \hat{\alpha}_1$ means that truth membership of $\hat{\alpha}_1$ is 0.451 and false membership of $\hat{\alpha}_1$ is 0.342. In other words, while selecting sub-parametric valued multi-argument tuple $\hat{\alpha}_1$, 45.1% experts have positive response and 34.2% have negative response.*

**Definition 5.** *An IFpHSS $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}$ is an IFpHS-subset of another IFpHSS $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$, denoted by $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\subseteq} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$ if $\hat{\Psi}_{\widetilde{\mathscr{B}}_1}(\hat{\alpha}) \leq \hat{\Psi}_{\widetilde{\mathscr{B}}_2}(\hat{\alpha}), \hat{\chi}_{\widetilde{\mathscr{B}}_1}(\hat{\alpha}) \geq \hat{\chi}_{\widetilde{\mathscr{B}}_2}(\hat{\alpha})$ and $\widehat{\psi}_{\widetilde{\mathscr{B}}_1}(\wp) \subseteq \widehat{\psi}_{\widetilde{\mathscr{B}}_2}(\wp)$ for all $\hat{\alpha} \in \widetilde{\mathscr{A}}$ and $\wp \in \widetilde{\mathscr{B}}$.*

**Example 3.** *Let $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}$ and $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$ be IFpHSS defined as*

$$\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} = \left\{ \begin{array}{l} \left(\langle 0.451, 0.342\rangle / \hat{\alpha}_1, \{\hat{\ell}_1, \hat{\ell}_3, \hat{\ell}_6\}\right), \left(\langle 0.557, 0.251\rangle / \hat{\alpha}_2, \{\hat{\ell}_2, \hat{\ell}_3, \hat{\ell}_7\}\right), \\ \left(\langle 0.781, 0.131\rangle / \hat{\alpha}_3, \{\hat{\ell}_4, \hat{\ell}_5, \hat{\ell}_8\}\right), \left(\langle 0.635, 0.311\rangle / \hat{\alpha}_4, \{\hat{\ell}_1, \hat{\ell}_7, \hat{\ell}_8\}\right) \end{array} \right\},$$

*and*

$$\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2} = \left\{ \begin{array}{l} \left(\langle 0.551, 0.242\rangle / \hat{\alpha}_1, \{\hat{\ell}_1, \hat{\ell}_3, \hat{\ell}_5, \hat{\ell}_6\}\right), \left(\langle 0.657, 0.151\rangle / \hat{\alpha}_2, \{\hat{\ell}_2, \hat{\ell}_3, \hat{\ell}_4, \hat{\ell}_7\}\right), \\ \left(\langle 0.811, 0.121\rangle / \hat{\alpha}_3, \{\hat{\ell}_4, \hat{\ell}_5, \hat{\ell}_6, \hat{\ell}_8\}\right), \left(\langle 0.735, 0.211\rangle / \hat{\alpha}_4, \{\hat{\ell}_1, \hat{\ell}_2, \hat{\ell}_7, \hat{\ell}_8\}\right) \end{array} \right\}.$$

*Then $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\subseteq} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$*

**Proposition 4.** *For IFpHSS $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}$, $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$, and $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_3}$, the results given below are true:*

1. *$\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\subseteq} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$ if and only if $\widetilde{\mathscr{B}}_1 \subseteq \widetilde{\mathscr{B}}_2$.*
2. *$\widetilde{\mathfrak{S}}_{\Phi} \tilde{\subseteq} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}$.*
3. *$\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\subseteq} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}$.*
4. *if $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\subseteq} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$ and $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2} \tilde{\subseteq} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_3}$ then $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\subseteq} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_3}$.*

**Definition 6.** *The IFpHSS $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}$ and $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$ are IFpHSS-equal, denoted by $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} = \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$, if and only if $\hat{\Psi}_{\widetilde{\mathscr{B}}_1}(\hat{\alpha}) = \hat{\Psi}_{\widetilde{\mathscr{B}}_2}(\hat{\alpha}), \hat{\chi}_{\widetilde{\mathscr{B}}_1}(\hat{\alpha}) = \hat{\chi}_{\widetilde{\mathscr{B}}_2}(\hat{\alpha})$ and $\widehat{\psi}_{\widetilde{\mathscr{B}}_1}(\wp)$, $\widehat{\psi}_{\widetilde{\mathscr{B}}_2}(\wp)$ have identical approximations for all $\hat{\alpha} \in \widetilde{\mathscr{A}}$ and $\wp \in \widetilde{\mathscr{B}}$.*

**Proposition 5.** *If $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}$, $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$, and $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_3}$ are IFpHSS, then results stated below are true:*

1. *if $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} = \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$ and $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2} = \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_3}$ then $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} = \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_3}$.*
2. *if $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\subseteq} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$ and $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2} \tilde{\subseteq} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \Leftrightarrow \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} = \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$.*

**Definition 7.** *The complement of an IFpHSS $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}}$, denoted by $\widetilde{\mathfrak{S}}^{\tilde{c}}_{\widetilde{\mathscr{B}}}$, is stated as $\hat{\Psi}^{\tilde{c}}_{\widetilde{\mathscr{B}}}(\hat{\alpha}) = \hat{\chi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}), \hat{\chi}^{\tilde{c}}_{\widetilde{\mathscr{B}}}(\hat{\alpha}) = \hat{\Psi}_{\widetilde{\mathscr{B}}}(\hat{\alpha})$ and $\widetilde{\mathfrak{S}}^{\tilde{c}}_{\widetilde{\mathscr{B}}}(\wp) = \widetilde{\mathscr{L}} \setminus \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}}(\wp)$ where $\setminus$ denotes set difference.*

**Example 6.** *Let $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}} \in \Theta_{IFPHS}(\widetilde{\mathscr{L}})$, and be given as*

$$\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}} = \left\{ \begin{array}{l} \left(\langle 0.451, 0.342\rangle / \hat{\alpha}_1, \{\hat{\ell}_1, \hat{\ell}_3, \hat{\ell}_6\}\right), \left(\langle 0.557, 0.251\rangle / \hat{\alpha}_2, \{\hat{\ell}_2, \hat{\ell}_3, \hat{\ell}_7\}\right), \\ \left(\langle 0.781, 0.131\rangle / \hat{\alpha}_3, \{\hat{\ell}_4, \hat{\ell}_5, \hat{\ell}_8\}\right), \left(\langle 0.635, 0.311\rangle / \hat{\alpha}_4, \{\hat{\ell}_1, \hat{\ell}_7, \hat{\ell}_8\}\right) \end{array} \right\},$$

*then $\widetilde{\mathfrak{S}}^{\tilde{c}}_{\widetilde{\mathscr{B}}}$ can be written as*

$$\widetilde{\mathfrak{S}}^{\tilde{c}}_{\widetilde{\mathscr{B}}} = \left\{ \begin{array}{l} \left(\langle 0.342, 0.451\rangle / \hat{\alpha}_1, \{\hat{\ell}_2, \hat{\ell}_4, \hat{\ell}_5, \hat{\ell}_7, \hat{\ell}_8\}\right), \left(\langle 0.251, 0.557\rangle / \hat{\alpha}_2, \{\hat{\ell}_1, \hat{\ell}_4, \hat{\ell}_5, \hat{\ell}_6, \hat{\ell}_8\}\right), \\ \left(\langle 0.131, 0.781\rangle / \hat{\alpha}_3, \{\hat{\ell}_1, \hat{\ell}_2, \hat{\ell}_3, \hat{\ell}_6, \hat{\ell}_7\}\right), \left(\langle 0.311, 0.635\rangle / \hat{\alpha}_4, \{\hat{\ell}_2, \hat{\ell}_3, \hat{\ell}_4, \hat{\ell}_5, \hat{\ell}_6\}\right) \end{array} \right\}.$$

**Proposition 7.** *For an IFpHSS $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}}$, we have*

1. $(\widetilde{\mathfrak{S}}^{\tilde{c}}_{\widetilde{\mathscr{B}}})^{\tilde{c}} = \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}}$.
2. $\widetilde{\mathfrak{S}}^{\tilde{c}}_{\phi} = \widetilde{\mathfrak{S}}_{\widetilde{\widetilde{\mathscr{B}}}}$.

**Definition 8.** *The union of IFpHSS $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}$ and $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$, denoted by $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\cup} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$, is stated as*

1. $\hat{\Psi}_{\widetilde{\mathscr{B}}_1 \tilde{\cup} \widetilde{\mathscr{B}}_2}(\hat{\alpha}) = max\{\hat{\Psi}_{\widetilde{\mathscr{B}}_1}(\hat{\alpha}), \hat{\Psi}_{\widetilde{\mathscr{B}}_2}(\hat{\alpha})\}$,
2. $\hat{\chi}_{\widetilde{\mathscr{B}}_1 \tilde{\cup} \widetilde{\mathscr{B}}_2}(\hat{\alpha}) = min\{\hat{\chi}_{\widetilde{\mathscr{B}}_1}(\hat{\alpha}), \hat{\chi}_{\widetilde{\mathscr{B}}_2}(\hat{\alpha})\}$,
3. $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1 \tilde{\cup} \widetilde{\mathscr{B}}_2}(\hat{\wp}) = \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}(\hat{\wp}) \cup \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}(\hat{\wp})$, *for all* $\hat{\alpha} \in \widetilde{\mathscr{A}}$ *and* $\hat{\wp} \in \widetilde{\mathscr{B}}$.

**Example 8.** *Let $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}$, and $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$ be IFpHSS stated as*

$$\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} = \left\{ \begin{array}{c} \left(\langle 0.451, 0.342\rangle/\hat{\alpha}_1, \{\hat{\ell}_1, \hat{\ell}_3, \hat{\ell}_6\}\right), \left(\langle 0.557, 0.251\rangle/\hat{\alpha}_2, \{\hat{\ell}_2, \hat{\ell}_3, \hat{\ell}_7\}\right), \\ \left(\langle 0.781, 0.131\rangle/\hat{\alpha}_3, \{\hat{\ell}_4, \hat{\ell}_5, \hat{\ell}_8\}\right), \left(\langle 0.635, 0.311\rangle/\hat{\alpha}_4, \{\hat{\ell}_1, \hat{\ell}_7, \hat{\ell}_8\}\right) \end{array} \right\},$$

*and*

$$\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2} = \left\{ \begin{array}{c} \left(\langle 0.551, 0.242\rangle/\hat{\alpha}_1, \{\hat{\ell}_1, \hat{\ell}_2, \hat{\ell}_6\}\right), \left(\langle 0.657, 0.151\rangle/\hat{\alpha}_2, \{\hat{\ell}_2, \hat{\ell}_4, \hat{\ell}_7\}\right), \\ \left(\langle 0.811, 0.121\rangle/\hat{\alpha}_3, \{\hat{\ell}_4, \hat{\ell}_6, \hat{\ell}_8\}\right), \left(\langle 0.735, 0.211\rangle/\hat{\alpha}_4, \{\hat{\ell}_1, \hat{\ell}_2, \hat{\ell}_8\}\right) \end{array} \right\}.$$

*Then $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\cup} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$ can written as*

$$\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\cup} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2} = \left\{ \begin{array}{c} \left(\langle 0.551, 0.242\rangle/\hat{\alpha}_1, \{\hat{\ell}_1, \hat{\ell}_2, \hat{\ell}_3, \hat{\ell}_6\}\right), \left(\langle 0.657, 0.151\rangle/\hat{\alpha}_2, \{\hat{\ell}_2, \hat{\ell}_3, \hat{\ell}_4, \hat{\ell}_7\}\right), \\ \left(\langle 0.811, 0.121\rangle/\hat{\alpha}_3, \{\hat{\ell}_4, \hat{\ell}_5, \hat{\ell}_6, \hat{\ell}_8\}\right), \left(\langle 0.735, 0.211\rangle/\hat{\alpha}_4, \{\hat{\ell}_1, \hat{\ell}_2, \hat{\ell}_7, \hat{\ell}_8\}\right) \end{array} \right\}.$$

**Proposition 9.** *If $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}$, $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$, and $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_3}$ are IFpHSS, then following results are valid:*

1. $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\cup} \widetilde{\mathfrak{S}}_{\Phi} = \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}$,
2. $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\cup} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2} = \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2} \tilde{\cup} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}$,
3. $(\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\cup} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}) \tilde{\cup} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_3} = \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\cup} (\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2} \tilde{\cup} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_3})$.

**Definition 9.** *The intersection of IFPHSS $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}$ and $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$, denoted by $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\cap} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$, is defined by*

(i) $\hat{\Psi}_{\widetilde{\mathscr{B}}_1 \tilde{\cap} \widetilde{\mathscr{B}}_2}(\hat{\alpha}) = min\{\hat{\Psi}_{\widetilde{\mathscr{B}}_1}(\hat{\alpha}), \hat{\Psi}_{\widetilde{\mathscr{B}}_2}(\hat{\alpha})\}$,

(ii) $\hat{\chi}_{\widetilde{\mathscr{B}}_1 \tilde{\cap} \widetilde{\mathscr{B}}_2}(\hat{\alpha}) = max\{\hat{\chi}_{\widetilde{\mathscr{B}}_1}(\hat{\alpha}), \hat{\chi}_{\widetilde{\mathscr{B}}_2}(\hat{\alpha})\}$,

(iii) $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1 \tilde{\cap} \widetilde{\mathscr{B}}_2}(\hat{\wp}) = \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}(\hat{\wp}) \cap \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}(\hat{\wp})$, *for all* $\hat{\alpha} \in \widetilde{\mathscr{A}}$ *and* $\hat{\wp} \in \widetilde{\mathscr{B}}$.

**Example 10.** *Let $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}$, and $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$ be IFpHSS defined as*

$$\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} = \left\{ \begin{array}{c} \left(\langle 0.451, 0.342\rangle/\hat{\alpha}_1, \{\hat{\ell}_1, \hat{\ell}_3, \hat{\ell}_6\}\right), \left(\langle 0.557, 0.251\rangle/\hat{\alpha}_2, \{\hat{\ell}_2, \hat{\ell}_3, \hat{\ell}_7\}\right), \\ \left(\langle 0.781, 0.131\rangle/\hat{\alpha}_3, \{\hat{\ell}_4, \hat{\ell}_5, \hat{\ell}_8\}\right), \left(\langle 0.635, 0.311\rangle/\hat{\alpha}_4, \{\hat{\ell}_1, \hat{\ell}_7, \hat{\ell}_8\}\right) \end{array} \right\},$$

*and*

$$\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2} = \left\{ \begin{array}{c} \left(\langle 0.551, 0.242\rangle/\hat{\alpha}_1, \{\hat{\ell}_1, \hat{\ell}_2, \hat{\ell}_6\}\right), \left(\langle 0.657, 0.151\rangle/\hat{\alpha}_2, \{\hat{\ell}_2, \hat{\ell}_4, \hat{\ell}_7\}\right), \\ \left(\langle 0.811, 0.121\rangle/\hat{\alpha}_3, \{\hat{\ell}_4, \hat{\ell}_6, \hat{\ell}_8\}\right), \left(\langle 0.735, 0.211\rangle/\hat{\alpha}_4, \{\hat{\ell}_1, \hat{\ell}_2, \hat{\ell}_8\}\right) \end{array} \right\}.$$

*Then* $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\cap} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$ *can written as*

$$\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\cap} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2} = \left\{ \begin{array}{c} \left(\langle 0.451, 0.342\rangle / \hat{\alpha}_1, \{\hat{\ell}_1, \hat{\ell}_6\}\right), \left(\langle 0.557, 0.251\rangle / \hat{\alpha}_2, \{\hat{\ell}_2, \hat{\ell}_7\}\right), \\ \left(\langle 0.781, 0.131\rangle / \hat{\alpha}_3, \{\hat{\ell}_4, \hat{\ell}_8\}\right), \left(\langle 0.635, 0.311\rangle / \hat{\alpha}_4, \{\hat{\ell}_1, \hat{\ell}_8\}\right) \end{array} \right\}.$$

**Proposition 11.** *If* $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}$, $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$, *and* $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_3}$ *are IFpHSS, then following results are valid:*

1. $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\cap} \widetilde{\mathfrak{S}}_{\Phi} = \widetilde{\mathfrak{S}}_{\Phi}$.
2. $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\cap} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2} = \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2} \tilde{\cap} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}$.
3. $(\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\cap} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}) \tilde{\cap} \widetilde{\mathfrak{S}}_{\mathfrak{S}_{\widetilde{\mathscr{B}}_3}} = \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\cap} (\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2} \tilde{\cap} \widetilde{\mathfrak{S}}_{\mathfrak{S}_{\widetilde{\mathscr{B}}_3}})$.

**Remark 12.** *For any IFpHSS* $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}}$, *if* $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}} \neq \widetilde{\mathfrak{S}}_{\tilde{G}}$, *then* $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}} \tilde{\cup} \widetilde{\mathfrak{S}}^{\tilde{c}}_{\widetilde{\mathscr{B}}} \neq \widetilde{\mathfrak{S}}_{\tilde{G}}$ *and* $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}} \tilde{\cap} \widetilde{\mathfrak{S}}^{\tilde{c}}_{\widetilde{\mathscr{B}}} \neq \widetilde{\mathfrak{S}}_{\Phi}$

**Proposition 13.** *Let* $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}, \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2} \in \Theta_{IFPHS}(\widetilde{\mathscr{L}})$ *then the following D'Morgan's laws are valid*

1. $(\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\cup} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2})^{\tilde{c}} = \widetilde{\mathfrak{S}}^{\tilde{c}}_{\widetilde{\mathscr{B}}_1} \tilde{\cap} \widetilde{\mathfrak{S}}^{\tilde{c}}_{\widetilde{\mathscr{B}}_2}$.
2. $(\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\cap} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2})^{\tilde{c}} = \widetilde{\mathfrak{S}}^{\tilde{c}}_{\widetilde{\mathscr{B}}_1} \tilde{\cup} \widetilde{\mathfrak{S}}^{\tilde{c}}_{\widetilde{\mathscr{B}}_2}$.

*Proof.* For all $\hat{\alpha} \in \widetilde{\mathscr{A}}$, and $\hat{\wp} \in \widetilde{\mathscr{B}}$,
(1). Since $(\hat{\Psi}_{\widetilde{\mathscr{B}}_1 \tilde{\cup} \widetilde{\mathscr{B}}_2})^{\tilde{c}}(\hat{\alpha}) = 1 - \hat{\Psi}_{\widetilde{\mathscr{B}}_1 \tilde{\cup} \widetilde{\mathscr{B}}_2}(\hat{\alpha}) = 1 - max\{\hat{\Psi}_{\widetilde{\mathscr{B}}_1}(\hat{\alpha}), \hat{\Psi}_{\widetilde{\mathscr{B}}_2}(\hat{\alpha})\}$,
$= min\{1 - \hat{\Psi}_{\widetilde{\mathscr{B}}_1}(\hat{\alpha}), 1 - \hat{\Psi}_{\widetilde{\mathscr{B}}_2}(\hat{\alpha})\} = min\{\hat{\Psi}^{\tilde{c}}_{\widetilde{\mathscr{B}}_1}(\hat{\alpha}), \hat{\Psi}^{\tilde{c}}_{\widetilde{\mathscr{B}}_2}(\hat{\alpha})\} = \hat{\Psi}^{\tilde{c}}_{\widetilde{\mathscr{B}}_1 \tilde{\cap} \widetilde{\mathscr{B}}_2}(\hat{\alpha})$,
and
$(\hat{\chi}_{\widetilde{\mathscr{B}}_1 \tilde{\cup} \widetilde{\mathscr{B}}_2})^{\tilde{c}}(\hat{\alpha}) = 1 - \hat{\chi}_{\widetilde{\mathscr{B}}_1 \tilde{\cup} \widetilde{\mathscr{B}}_2}(\hat{\alpha}) = 1 - min\{\hat{\chi}_{\widetilde{\mathscr{B}}_1}(\hat{\alpha}), \hat{\chi}_{\widetilde{\mathscr{B}}_2}(\hat{\alpha})\}$,
$= max\{1 - \hat{\chi}_{\widetilde{\mathscr{B}}_1}(\hat{\alpha}), 1 - \hat{\chi}_{\widetilde{\mathscr{B}}_2}(\hat{\alpha})\} = max\{\hat{\chi}^{\tilde{c}}_{\widetilde{\mathscr{B}}_1}(\hat{\alpha}), \hat{\chi}^{\tilde{c}}_{\widetilde{\mathscr{B}}_2}(\hat{\alpha})\} = \hat{\chi}^{\tilde{c}}_{\widetilde{\mathscr{B}}_1 \tilde{\cap} \widetilde{\mathscr{B}}_2}(\hat{\alpha})$,
and
$(\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1 \tilde{\cup} \widetilde{\mathscr{B}}_2})^{\tilde{c}}(\hat{\wp}) = \widetilde{\mathscr{L}} \setminus \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1 \tilde{\cup} \widetilde{\mathscr{B}}_2}(\hat{\wp}) = \widetilde{\mathscr{L}} \setminus (\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}(\hat{\wp}) \cup \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}(\hat{\wp}))$,
$= (\widetilde{\mathscr{L}} \setminus \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}(\hat{\wp})) \cap (\widetilde{\mathscr{L}} \setminus \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}(\hat{\wp})) = \widetilde{\mathfrak{S}}^{\tilde{c}}_{\widetilde{\mathscr{B}}_1}(\hat{\wp}) \tilde{\cap} \widetilde{\mathfrak{S}}^{\tilde{c}}_{\widetilde{\mathscr{B}}_2}(\hat{\wp}) = \widetilde{\mathfrak{S}}^{\tilde{c}}_{\widetilde{\mathscr{B}}_1 \tilde{\cap} \widetilde{\mathscr{B}}_2}(\hat{\wp})$.
Similarly (2) can be proved easily. □

**Proposition 14.** *If* $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}$, $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$, *and* $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_3}$ *are IFpHSS, then following results are valid:*

1. $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\cup} (\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2} \tilde{\cap} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_3}) = (\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\cup} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}) \tilde{\cap} (\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\cup} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_3})$.
2. $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\cap} (\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2} \tilde{\cup} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_3}) = (\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\cap} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}) \tilde{\cup} (\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\cap} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_3})$.

*Proof.* For all $\hat{\alpha} \in \widetilde{\mathscr{A}}$,
(1). Since $\hat{\Psi}_{\widetilde{\mathscr{B}}_1 \tilde{\cup} (\widetilde{\mathscr{B}}_2 \tilde{\cap} \widetilde{\mathscr{B}}_3)}(\hat{\alpha}) = max\{\hat{\Psi}_{\widetilde{\mathscr{B}}_1}(\hat{\alpha}), \hat{\Psi}_{\widetilde{\mathscr{B}}_2 \tilde{\cap} \widetilde{\mathscr{B}}_3}(\hat{\alpha})\}$
$= max\{\hat{\Psi}_{\widetilde{\mathscr{B}}_1}(\hat{\alpha}), min\{\hat{\Psi}_{\widetilde{\mathscr{B}}_2}(\hat{\alpha}), \hat{\Psi}_{\widetilde{\mathscr{B}}_3}(\hat{\alpha})\}\}$
$= min\{max\{\hat{\Psi}_{\widetilde{\mathscr{B}}_1}(\hat{\alpha}), \hat{\Psi}_{\widetilde{\mathscr{B}}_2}(\hat{\alpha})\}, max\{\hat{\Psi}_{\widetilde{\mathscr{B}}_1}(\hat{\alpha}), \hat{\Psi}_{\widetilde{\mathscr{B}}_3}(\hat{\alpha})\}\}$
$= min\{\hat{\Psi}_{\widetilde{\mathscr{B}}_1 \tilde{\cup} \widetilde{\mathscr{B}}_2}(\hat{\alpha}), \hat{\Psi}_{\widetilde{\mathscr{B}}_1 \tilde{\cup} \widetilde{\mathscr{B}}_3}(\hat{\alpha})\}$
$= \hat{\Psi}_{(\widetilde{\mathscr{B}}_1 \tilde{\cup} \widetilde{\mathscr{B}}_2) \tilde{\cap} (\widetilde{\mathscr{B}}_1 \tilde{\cup} \widetilde{\mathscr{B}}_3)}(\hat{\alpha})$
and
$\hat{\chi}_{\widetilde{\mathscr{B}}_1 \tilde{\cup} (\widetilde{\mathscr{B}}_2 \tilde{\cap} \widetilde{\mathscr{B}}_3)}(\hat{\alpha}) = min\{\hat{\chi}_{\widetilde{\mathscr{B}}_1}(\hat{\alpha}), \hat{\chi}_{\widetilde{\mathscr{B}}_2 \tilde{\cap} \widetilde{\mathscr{B}}_3}(\hat{\alpha})\}$
$= min\{\hat{\chi}_{\widetilde{\mathscr{B}}_1}(\hat{\alpha}), max\{\hat{\chi}_{\widetilde{\mathscr{B}}_2}(\hat{\alpha}), \hat{\chi}_{\widetilde{\mathscr{B}}_3}(\hat{\alpha})\}\}$
$= max\{min\{\hat{\chi}_{\widetilde{\mathscr{B}}_1}(\hat{\alpha}), \hat{\chi}_{\widetilde{\mathscr{B}}_2}(\hat{\alpha})\}, min\{\hat{\chi}_{\widetilde{\mathscr{B}}_1}(\hat{\alpha}), \hat{\chi}_{\widetilde{\mathscr{B}}_3}(\hat{\alpha})\}\}$
$= max\{\hat{\chi}_{\widetilde{\mathscr{B}}_1 \tilde{\cup} \widetilde{\mathscr{B}}_2}(\hat{\alpha}), \hat{\chi}_{\widetilde{\mathscr{B}}_1 \tilde{\cup} \widetilde{\mathscr{B}}_3}(\hat{\alpha})\}$

$= \hat{\chi}_{(\widetilde{\mathscr{B}}_1 \tilde{\cup} \widetilde{\mathscr{B}}_2) \tilde{\cap} (\widetilde{\mathscr{B}}_1 \tilde{\cup} \widetilde{\mathscr{B}}_3)}(\hat{\alpha})$
and
$\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1 \tilde{\cup} (\widetilde{\mathscr{B}}_2 \tilde{\cap} \widetilde{\mathscr{B}}_3)}(\hat{\wp}) = \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}(\hat{\wp}) \cup \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2 \tilde{\cap} \widetilde{\mathscr{B}}_3}(\hat{\wp})$
$= \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}(\hat{\wp}) \cup (\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}(\hat{\wp}) \cap \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_3}(\hat{\wp}))$
$= (\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}(\hat{\wp}) \cup \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}(\hat{\wp})) \cap (\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}(\hat{\wp}) \cup \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_3}(\hat{\wp}))$
$= \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1 \tilde{\cup} \widetilde{\mathscr{B}}_2}(\hat{\wp}) \cap \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1 \tilde{\cup} \widetilde{\mathscr{B}}_3}(\hat{\wp})$
$= \widetilde{\mathfrak{S}}_{(\widetilde{\mathscr{B}}_1 \tilde{\cup} \widetilde{\mathscr{B}}_2) \tilde{\cap} (\widetilde{\mathscr{B}}_1 \tilde{\cup} \widetilde{\mathscr{B}}_3)}(\hat{\wp})$
In the same way, (2) can be proved. □

**Definition 10.** *The OR-operation $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\oplus} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$ of IFpHSS $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}$ and $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$, is stated as*

(i) $\hat{\Psi}_{\widetilde{\mathscr{B}}_1 \tilde{\oplus} \widetilde{\mathscr{B}}_2}(\hat{\alpha}_1, \hat{\alpha}_2) = max\{\hat{\Psi}_{\widetilde{\mathscr{B}}_1}(\hat{\alpha}_1), \hat{\Psi}_{\widetilde{\mathscr{B}}_2}(\hat{\alpha}_2)\}$,

(ii) $\hat{\chi}_{\widetilde{\mathscr{B}}_1 \tilde{\oplus} \widetilde{\mathscr{B}}_2}(\hat{\alpha}_1, \hat{\alpha}_2) = min\{\hat{\chi}_{\widetilde{\mathscr{B}}_1}(\hat{\alpha}_1), \hat{\chi}_{\widetilde{\mathscr{B}}_2}(\hat{\alpha}_2)\}$,

(iii) $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1 \tilde{\oplus} \widetilde{\mathscr{B}}_2}(\hat{\wp}_1, \hat{\wp}_2) = \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}(\hat{\wp}_1) \cup \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}(\hat{\wp}_2)$,

*for all* $(\hat{\alpha}_1, \hat{\alpha}_2) \in \widetilde{\mathscr{A}_1} \times \widetilde{\mathscr{A}_2}$ *and* $(\hat{\wp}_1, \hat{\wp}_2) \in \widetilde{\mathscr{B}}_1 \times \widetilde{\mathscr{B}}_2$.

**Definition 11.** *The AND-operation $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\otimes} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$ of IFpHSS $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}$ and $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$, is stated as*

(i) $\hat{\Psi}_{\widetilde{\mathscr{B}}_1 \tilde{\otimes} \widetilde{\mathscr{B}}_2}(\hat{\alpha}_1, \hat{\alpha}_2) = min\{\hat{\Psi}_{\widetilde{\mathscr{B}}_1}(\hat{\alpha}_1), \hat{\Psi}_{\widetilde{\mathscr{B}}_2}(\hat{\alpha}_2)\}$,

(ii) $\hat{\chi}_{\widetilde{\mathscr{B}}_1 \tilde{\otimes} \widetilde{\mathscr{B}}_2}(\hat{\alpha}_1, \hat{\alpha}_2) = max\{\hat{\chi}_{\widetilde{\mathscr{B}}_1}(\hat{\alpha}_1), \hat{\chi}_{\widetilde{\mathscr{B}}_2}(\hat{\alpha}_2)\}$,

(iii) $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1 \tilde{\otimes} \widetilde{\mathscr{B}}_2}(\hat{\wp}_1, \hat{\wp}_2) = \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}(\hat{\wp}_1) \cap \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}(\hat{\wp}_2)$,

*for all* $(\hat{\alpha}_1, \hat{\alpha}_2) \in \widetilde{\mathscr{A}_1} \times \widetilde{\mathscr{A}_2}$ *and* $(\hat{\wp}_1, \hat{\wp}_2) \in \widetilde{\mathscr{B}}_1 \times \widetilde{\mathscr{B}}_2$.

**Proposition 15.** *For IFpHSS $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1}$, $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}$, and $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_3}$, following results are true:*

1. $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\otimes} \widetilde{\mathfrak{S}}_{\Phi} = \widetilde{\mathfrak{S}}_{\Phi}$.
2. $(\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\otimes} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}) \tilde{\otimes} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_3} = \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\otimes} (\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2} \tilde{\otimes} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_3})$.
3. $(\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\oplus} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2}) \tilde{\oplus} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_3} = \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_1} \tilde{\oplus} (\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_2} \tilde{\oplus} \widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}_3})$.

## 4. INTUITIONISTIC FUZZY DECISION SET OF IFPHSS

Motivated by the decision-making methodologies discussed in references [19–25], this study introduces a novel algorithm that leverages the characterization of the intuitionistic fuzzy decision set within the framework of IFpHSS. The proposed algorithm is grounded in a structured decision-making technique that effectively handles complex uncertainty and multi-attribute evaluation. To demonstrate the practical applicability and effectiveness of the approach, a prototype case study is provided, illustrating the step-by-step implementation of the algorithm in a real-world decision-making context.

**Definition 12.** *Let $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}} \in \Theta_{IFPHS}(\widetilde{\mathscr{L}})$ then a intuitionistic fuzzy decision set of $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}}$ (i.e. $\widetilde{\mathfrak{S}}^{\mathcal{D}}_{\widetilde{\mathscr{B}}}$) is represented as*

$$\widetilde{\mathfrak{S}}^{\mathcal{D}}_{\widetilde{\mathscr{B}}} = \left\{ \langle \hat{\Psi}^{\mathcal{D}}_{\widetilde{\mathscr{B}}}(\hat{\ell}), \hat{\chi}^{\mathcal{D}}_{\widetilde{\mathscr{B}}}(\hat{\ell}) \rangle / \hat{\ell} : \hat{\ell} \in \widetilde{\mathscr{L}} \right\}$$

*where* $\hat{\Psi}^{\mathcal{D}}_{\widetilde{\mathscr{B}}}, \hat{\chi}^{\mathcal{D}}_{\widetilde{\mathscr{B}}} : \widetilde{\mathscr{L}} \to [0,1]$ *and*

$$\hat{\Psi}^{\mathcal{D}}_{\widetilde{\mathscr{B}}}(\hat{\ell}) = \frac{1}{\left|\widetilde{\mathscr{L}}\right|} \sum_{\hat{\hbar} \in S(\widetilde{\mathscr{B}})} \hat{\Psi}_{\widetilde{\mathscr{B}}}(\hat{\hbar}) \hat{\Xi}_{\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}}(\hat{\hbar})}(\hat{\ell})$$

$$\hat{\chi}^{\mathcal{D}}_{\widetilde{\mathscr{B}}}(\hat{\ell}) = \frac{1}{\left|\widetilde{\mathscr{L}}\right|} \sum_{\hat{\hbar} \in \mathbb{S}(\widetilde{\mathscr{B}})} \hat{\chi}_{\widetilde{\mathscr{B}}}(\hat{\hbar}) \hat{\Xi}_{\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}}(\hat{\hbar})}(\hat{\ell})$$

*where* $\mathbb{S}(\widetilde{\mathscr{B}})$ *is support of* $\widetilde{\mathscr{B}}$ *and* $\left|\widetilde{\mathscr{L}}\right|$ *denotes the number of elements in* $\widetilde{\mathscr{L}}$ *with*

$$\hat{\Xi}_{\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}}(\hat{\hbar})}(\hat{\ell}) = \begin{cases} 1 & ; \quad \hat{\ell} \in \hat{\Xi}_{\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}}}(\hat{\hbar}) \\ 0 & ; \quad \hat{\ell} \notin \hat{\Xi}_{\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}}}(\hat{\hbar}) \end{cases}$$

**Definition 13.** *If* $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}} \in \Theta_{IFPHS}(\widetilde{\mathscr{L}})$ *with intuitionistic fuzzy decision set* $\widetilde{\mathfrak{S}}^{\mathcal{D}}_{\widetilde{\mathscr{B}}}$ *then reduced fuzzy set of* $\widetilde{\mathfrak{S}}^{\mathcal{D}}_{\widetilde{\mathscr{B}}}$ *is a fuzzy set represented as*

$$\mathbb{R}(\widetilde{\mathfrak{S}}^{\mathcal{D}}_{\widetilde{\mathscr{B}}}) = \left\{ \zeta_{\widetilde{\mathfrak{S}}^{\mathcal{D}}_{\widetilde{\mathscr{B}}}}(\hat{\ell})/\hat{\ell} : \hat{\ell} \in \widetilde{\mathscr{L}} \right\}$$

*where* $\zeta_{\widetilde{\mathfrak{S}}^{\mathcal{D}}_{\widetilde{\mathscr{B}}}} : \widetilde{\mathscr{L}} \to [0,1]$ *with* $\zeta_{\widetilde{\mathfrak{S}}^{\mathcal{D}}_{\widetilde{\mathscr{B}}}}(\hat{\ell}) = \hat{\Psi}^{\mathcal{D}}_{\widetilde{\mathscr{B}}}(\hat{\ell})(1 - \hat{\chi}^{\mathcal{D}}_{\widetilde{\mathscr{B}}}(\hat{\ell}))$

### 4.1. PROBLEM STATEMENT

An educationist intends to establish a new educational institution and is deeply concerned about ensuring both demographic diversity and avoiding areas already saturated with similar institutions. His vision is to create an inclusive, forward-thinking learning environment that caters to students from varied social, cultural, and economic backgrounds. To make an informed and strategic decision, he has engaged a team of experts in urban planning, educational development, and demographic analysis. These professionals are tasked with conducting a thorough assessment of potential locations based on key indicators such as accessibility, population composition, socio-economic diversity, proximity to existing institutions, infrastructural readiness, and long-term growth potential. The ultimate goal is to identify a location that not only minimizes competition and redundancy but also maximizes outreach, community impact, and sustainability, thereby laying a strong foundation for academic excellence and social equity.

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**Algorithm 1.** *To select an appropriate site for educational institution, the following robust algorithm is proposed. This algorithm is the modified version of Algorithm presented by Çağman and Karataş [21].*

1. *Based on multi-argument tuples* $\hat{\alpha}$ *contained in* $\widetilde{\mathscr{A}}$*, construct an intuitionistic fuzzy set* $\widetilde{\mathscr{B}} = \{\langle \hat{\Psi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}), \hat{\chi}_{\widetilde{\mathscr{B}}}(\hat{\alpha})\rangle/\hat{\alpha} : \hat{\Psi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}), \hat{\chi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}) \in [0,1], \hat{\alpha} \in \widetilde{\mathscr{A}}\}$ *over* $\widetilde{\mathscr{A}}$,
2. *For all* $\hat{\alpha} \in \widetilde{\mathscr{A}}$*, compute their respective approximations* $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}}(\hat{\alpha})$,
3. *Using step (1) and step (2), construct IFpHSS* $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}}$ *over* $\widetilde{\mathscr{L}}$,
4. *Using formulations of Definition 12, compute* $\widetilde{\mathfrak{S}}^{\mathcal{D}}_{\widetilde{\mathscr{B}}}$ ,
5. *Determine reduced fuzzy sets using the formulations of Definition 13,*
6. *Choose the maximum of* $\zeta_{\widetilde{\mathfrak{S}}^{\mathcal{D}}_{\widetilde{\mathscr{B}}}}(\hat{\ell})$.

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**Example 16.** *Suppose Mrs. Smith is an educationist who wants to establish an educational institution to provide quality education to children in the nearby area. Other institutions in the populated region face various issues, including transportation problems. However, she believes that the location is crucial when establishing any institution. Therefore, she shortlisted eight sites for her new institution. These sites form the set of alternatives.* $\widetilde{\mathscr{L}} = \{\hat{\ell}_1, \hat{\ell}_2, \hat{\ell}_3, \hat{\ell}_4, \hat{\ell}_5, \hat{\ell}_6, \hat{\ell}_7, \hat{\ell}_8\}$. *Now, only one site is to be chosen for the institution. She hired some experts to help her evaluate the sites. The experts considered the parameters like* $\hat{æ}_1$ *= Accessibility,* $\hat{æ}_2$ =

Table 1. Tabulation of truth grades $\hat{\Psi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_i)$

| $\hat{\Psi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_i)$ | Degree | $\hat{\Psi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_i)$ | Degree |
|---|---|---|---|
| $\hat{\Psi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_1)$ | 0.1 | $\hat{\Psi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_9)$ | 0.9 |
| $\hat{\Psi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_2)$ | 0.2 | $\hat{\Psi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_{10})$ | 0.16 |
| $\hat{\Psi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_3)$ | 0.3 | $\hat{\Psi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_{11})$ | 0.25 |
| $\hat{\Psi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_4)$ | 0.4 | $\hat{\Psi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_{12})$ | 0.45 |
| $\hat{\Psi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_5)$ | 0.5 | $\hat{\Psi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_{13})$ | 0.35 |
| $\hat{\Psi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_6)$ | 0.6 | $\hat{\Psi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_{14})$ | 0.75 |
| $\hat{\Psi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_7)$ | 0.7 | $\hat{\Psi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_{15})$ | 0.65 |
| $\hat{\Psi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_8)$ | 0.8 | $\hat{\Psi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_{16})$ | 0.85 |

*Safety, $\hat{æ}_3$ = Environment, $\hat{æ}_4$ = Economic factors, and $\hat{æ}_5$ = Socio-cultural factors for this evaluation. These parameters are further classified in terms of their respective sub-parametric values that are enclosed in the sets $\widetilde{\mathscr{A}_1} = \{\hat{æ}_{11}, \hat{æ}_{12}\}$, $\widetilde{\mathscr{A}_2} = \{\hat{æ}_{21}, \hat{æ}_{22}\}$, $\widetilde{\mathscr{A}_3} = \{\hat{æ}_{31}, \hat{æ}_{32}\}$, $\widetilde{\mathscr{A}_4} = \{\hat{æ}_{41}, \hat{æ}_{42}\}$ and $\widetilde{\mathscr{A}_5} = \{\hat{æ}_{51}\}$, where $\hat{æ}_{11}$ = Proximity to residential areas, $\hat{æ}_{12}$ = Availability of public transportation, $\hat{æ}_{21}$ = Low crime rate in the area, $\hat{æ}_{22}$ = Presence of police stations nearby, $\hat{æ}_{31}$ = Low pollution, $\hat{æ}_{32}$ = Away from industrial or high-traffic zones, $\hat{æ}_{41}$ = Affordability of land or rent, $\hat{æ}_{42}$ = Proximity to shops, hostels, or food services, and $\hat{æ}_{51}$ = Community openness to education then $B = \widetilde{\mathscr{A}_1} \times \widetilde{\mathscr{A}_2} \times \widetilde{\mathscr{A}_3} \times \widetilde{\mathscr{A}_4} \times \widetilde{\mathscr{A}_5} = \{\hat{\alpha}_1, \hat{\alpha}_2, \hat{\alpha}_3, \hat{\alpha}_4, ..., \hat{\alpha}_{16}\}$ where each $\hat{\alpha}_i, i = 1, 2, ..., 16$, is a multi-argument tuple representing five different argument simultaneously. From tables 1, 2, we can construct $\widetilde{\mathscr{B}}$ as*

$$\widetilde{\mathscr{B}} = \left\{ \begin{array}{l} \wp_1 = \langle 0.1, 0.3\rangle / \hat{\alpha}_1, \wp_2 = \langle 0.2, 0.4\rangle / \hat{\alpha}_2, \wp_3 = \langle 0.3, 0.5\rangle / \hat{\alpha}_3, \wp_4 = \langle 0.4, 0.6\rangle / \hat{\alpha}_4, \\ \wp_5 = \langle 0.5, 0.7\rangle / \hat{\alpha}_5, \wp_6 = \langle 0.6, 0.8\rangle / \hat{\alpha}_6, \wp_7 = \langle 0.7, 0.9\rangle / \hat{\alpha}_7, \wp_8 = \langle 0.8, 0.1\rangle / \hat{\alpha}_8, \\ \wp_9 = \langle 0.9, 0.2\rangle / \hat{\alpha}_9, \wp_{10} = \langle 0.16, 0.37\rangle / \hat{\alpha}_{10}, \wp_{11} = \langle 0.25, 0.45\rangle / \hat{\alpha}_{11}, \wp_{12} = \langle 0.45, 0.65\rangle / \hat{\alpha}_{12}, \\ \wp_{13} = \langle 0.35, 0.55\rangle / \hat{\alpha}_{13}, \wp_{14} = \langle 0.75, 0.95\rangle / \hat{\alpha}_{14}, \wp_{15} = \langle 0.65, 0.85\rangle / \hat{\alpha}_{15}, \wp_{16} = \langle 0.85, 0.96\rangle / \hat{\alpha}_{16} \end{array} \right\}.$$

*Table 3 presents $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}}(\wp_i)$ corresponding to each element of $\widetilde{\mathscr{B}}$. Now, based on previous steps, an IFpHSS $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}}$ is constructed as*

$$\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}} = \left\{ \begin{array}{l} \left(\langle 0.1, 0.3\rangle / \hat{\alpha}_1, \{\hat{\ell}_1, \hat{\ell}_2\}\right), \left(\langle 0.2, 0.4\rangle / \hat{\alpha}_2, \{\hat{\ell}_1, \hat{\ell}_2, \hat{\ell}_3\}\right), \\ \left(\langle 0.3, 0.5\rangle / \hat{\alpha}_3, \{\hat{\ell}_2, \hat{\ell}_3, \hat{\ell}_4\}\right), \left(\langle 0.4, 0.6\rangle / \hat{\alpha}_4, \{\hat{\ell}_4, \hat{\ell}_5, \hat{\ell}_6\}\right), \\ \left(\langle 0.5, 0.7\rangle / \hat{\alpha}_5, \{\hat{\ell}_6, \hat{\ell}_7, \hat{\ell}_8\}\right), \left(\langle 0.6, 0.8\rangle / \hat{\alpha}_6, \{\hat{\ell}_2, \hat{\ell}_3, \hat{\ell}_4\}\right), \\ \left(\langle 0.7, 0.9\rangle / \hat{\alpha}_7, \{\hat{\ell}_1, \hat{\ell}_3, \hat{\ell}_5\}\right), \left(\langle 0.8, 0.1\rangle / \hat{\alpha}_8, \{\hat{\ell}_2, \hat{\ell}_3, \hat{\ell}_7\}\right), \\ \left(\langle 0.9, 0.2\rangle / \hat{\alpha}_9, \{\hat{\ell}_2, \hat{\ell}_7, \hat{\ell}_8\}\right), \left(\langle 0.16, 0.37\rangle / \hat{\alpha}_{10}, \{\hat{\ell}_6, \hat{\ell}_7, \hat{\ell}_8\}\right), \\ \left(\langle 0.25, 0.45\rangle / \hat{\alpha}_{11}, \{\hat{\ell}_2, \hat{\ell}_4, \hat{\ell}_6\}\right), \left(\langle 0.45, 0.65\rangle / \hat{\alpha}_{12}, \{\hat{\ell}_2, \hat{\ell}_3, \hat{\ell}_6\}\right), \\ \left(\langle 0.35, 0.55\rangle / \hat{\alpha}_{13}, \{\hat{\ell}_3, \hat{\ell}_5, \hat{\ell}_7\}\right), \left(\langle 0.75, 0.95\rangle / \hat{\alpha}_{14}, \{\hat{\ell}_1, \hat{\ell}_3, \hat{\ell}_5\}\right), \\ \left(\langle 0.65, 0.85\rangle / \hat{\alpha}_{15}, \{\hat{\ell}_5, \hat{\ell}_7, \hat{\ell}_8\}\right), \left(\langle 0.85, 0.96\rangle / \hat{\alpha}_{16}, \{\hat{\ell}_4, \hat{\ell}_5, \hat{\ell}_6\}\right) \end{array} \right\}.$$

*From Tables 4, 5 and 6 , the reduced fuzzy set $\mathbb{R}(\widetilde{\mathfrak{S}}^{\mathcal{D}}_{\widetilde{\mathscr{B}}})$ is constructed as*

Table 2. Tabulation of False grades $\hat{\chi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_i)$

| $\hat{\chi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_i)$ | Degree | $\hat{\chi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_i)$ | Degree |
|---|---|---|---|
| $\hat{\chi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_1)$ | 0.3 | $\hat{\chi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_9)$ | 0.2 |
| $\hat{\chi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_2)$ | 0.4 | $\hat{\chi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_{10})$ | 0.37 |
| $\hat{\chi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_3)$ | 0.5 | $\hat{\chi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_{11})$ | 0.45 |
| $\hat{\chi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_4)$ | 0.6 | $\hat{\chi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_{12})$ | 0.65 |
| $\hat{\chi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_5)$ | 0.7 | $\hat{\chi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_{13})$ | 0.55 |
| $\hat{\chi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_6)$ | 0.8 | $\hat{\chi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_{14})$ | 0.95 |
| $\hat{\chi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_7)$ | 0.9 | $\hat{\chi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_{15})$ | 0.85 |
| $\hat{\chi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_8)$ | 0.1 | $\hat{\chi}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_{16})$ | 0.96 |

Table 3. Approximate functions $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_i)$

| $\hat{\alpha}_i$ | $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_i)$ | $\hat{\alpha}_i$ | $\widetilde{\mathfrak{S}}_{\widetilde{\mathscr{B}}}(\hat{\alpha}_i)$ |
|---|---|---|---|
| $\hat{\alpha}_1$ | $\{\hat{\ell}_1, \hat{\ell}_2\}$ | $\hat{\alpha}_9$ | $\{\hat{\ell}_2, \hat{\ell}_7, \hat{\ell}_8\}$ |
| $\hat{\alpha}_2$ | $\{\hat{\ell}_1, \hat{\ell}_2, \hat{\ell}_3\}$ | $\hat{\alpha}_{10}$ | $\{\hat{\ell}_6, \hat{\ell}_7, \hat{\ell}_8\}$ |
| $\hat{\alpha}_3$ | $\{\hat{\ell}_2, \hat{\ell}_3, \hat{\ell}_4\}$ | $\hat{\alpha}_{11}$ | $\{\hat{\ell}_2, \hat{\ell}_4, \hat{\ell}_6\}$ |
| $\hat{\alpha}_4$ | $\{\hat{\ell}_4, \hat{\ell}_5, \hat{\ell}_6\}$ | $\hat{\alpha}_{12}$ | $\{\hat{\ell}_2, \hat{\ell}_3, \hat{\ell}_6\}$ |
| $\hat{\alpha}_5$ | $\{\hat{\ell}_6, \hat{\ell}_7, \hat{\ell}_8\}$ | $\hat{\alpha}_{13}$ | $\{\hat{\ell}_3, \hat{\ell}_5, \hat{\ell}_7\}$ |
| $\hat{\alpha}_6$ | $\{\hat{\ell}_2, \hat{\ell}_3, \hat{\ell}_4\}$ | $\hat{\alpha}_{14}$ | $\{\hat{\ell}_1, \hat{\ell}_3, \hat{\ell}_5\}$ |
| $\hat{\alpha}_7$ | $\{\hat{\ell}_1, \hat{\ell}_3, \hat{\ell}_5\}$ | $\hat{\alpha}_{15}$ | $\{\hat{\ell}_5, \hat{\ell}_7, \hat{\ell}_8\}$ |
| $\hat{\alpha}_8$ | $\{\hat{\ell}_2, \hat{\ell}_3, \hat{\ell}_7\}$ | $\hat{\alpha}_{16}$ | $\{\hat{\ell}_4, \hat{\ell}_5, \hat{\ell}_6\}$ |

Table 4. Membership values $\hat{\Psi}^D_{\widetilde{\mathscr{B}}}(\hat{\ell}_i)$

| $\hat{\ell}_i$ | $\hat{\Psi}^D_{\widetilde{\mathscr{B}}}(\hat{\ell}_i)$ | $\hat{\ell}_i$ | $\hat{\Psi}^D_{\widetilde{\mathscr{B}}}(\hat{\ell}_i)$ |
|---|---|---|---|
| $\hat{\ell}_1$ | 0.2188 | $\hat{\ell}_5$ | 0.4625 |
| $\hat{\ell}_2$ | 0.4500 | $\hat{\ell}_6$ | 0.3263 |
| $\hat{\ell}_3$ | 0.5188 | $\hat{\ell}_7$ | 0.4200 |
| $\hat{\ell}_4$ | 0.3000 | $\hat{\ell}_8$ | 0.2763 |

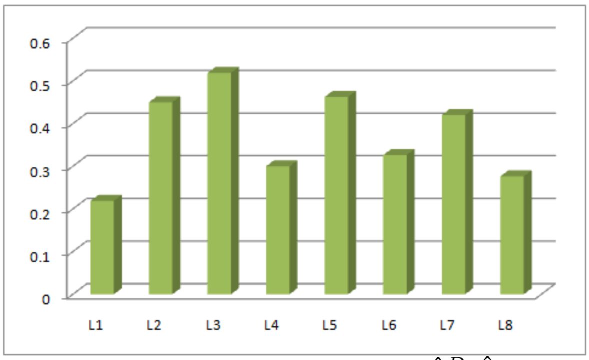


Figure 1. Membership values $\hat{\Psi}^{D}_{\widetilde{\mathscr{B}}}(\hat{\ell}_i)$

Table 5. Non-Membership values $\hat{\chi}^{D}_{\widetilde{\mathscr{B}}}(\hat{\ell}_i)$

| $\hat{\ell}_i$ | $\hat{\chi}^{D}_{\widetilde{\mathscr{B}}}(\hat{\ell}_i)$ | $\hat{\ell}_i$ | $\hat{\chi}^{D}_{\widetilde{\mathscr{B}}}(\hat{\ell}_i)$ |
|---|---|---|---|
| $\hat{\ell}_1$ | 0.3188 | $\hat{\ell}_5$ | 0.6013 |
| $\hat{\ell}_2$ | 0.4250 | $\hat{\ell}_6$ | 0.4663 |
| $\hat{\ell}_3$ | 0.6063 | $\hat{\ell}_7$ | 0.3463 |
| $\hat{\ell}_4$ | 0.4138 | $\hat{\ell}_8$ | 0.2650 |

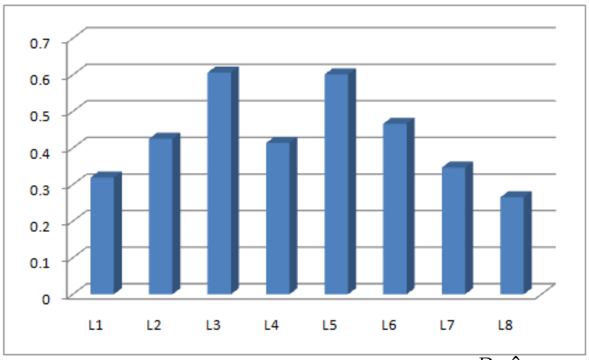


Figure 2. Non-Membership values $\hat{\chi}^{D}_{\widetilde{\mathscr{B}}}(\hat{\ell}_i)$

Table 6. Reduced Fuzzy membership $\zeta_{\widetilde{\mathfrak{S}}^{D}_{\widetilde{\mathscr{B}}}}(\hat{\ell}_i)$

| $\hat{\ell}_i$ | $\zeta_{\widetilde{\mathfrak{S}}^{D}_{\widetilde{\mathscr{B}}}}(\hat{\ell}_i)$ | $\hat{\ell}_i$ | $\zeta_{\widetilde{\mathfrak{S}}^{D}_{\widetilde{\mathscr{B}}}}(\hat{\ell}_i)$ |
|---|---|---|---|
| $\hat{\ell}_1$ | 0.1490 | $\hat{\ell}_5$ | 0.1844 |
| $\hat{\ell}_2$ | 0.2588 | $\hat{\ell}_6$ | 0.1741 |
| $\hat{\ell}_3$ | 0.2043 | $\hat{\ell}_7$ | 0.2746 |
| $\hat{\ell}_4$ | 0.1759 | $\hat{\ell}_8$ | 0.2030 |

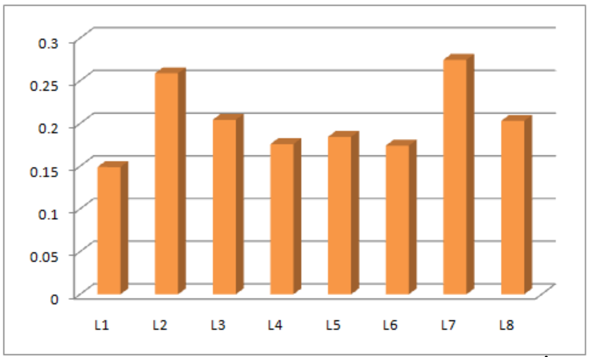


Figure 3. Reduced Fuzzy membership $\zeta_{\widetilde{\mathfrak{S}}^{\mathcal{D}}_{\widetilde{\mathscr{B}}}}(\hat{\ell}_i)$

$\mathbb{R}(\widetilde{\mathfrak{S}}^{\mathcal{D}}_{\widetilde{\mathscr{B}}}) = \left\{ \begin{array}{c} 0.1490/\hat{\ell}_1, 0.2588/\hat{\ell}_2, 0.2043/\hat{\ell}_3, 0.1759/\hat{\ell}_4, \\ 0.1844/\hat{\ell}_5, 0.1741/\hat{\ell}_6, 0.2746/\hat{\ell}_7, 0.2030/\hat{\ell}_8 \end{array} \right\}$. *Since maximum of* $\zeta_{\widetilde{\mathfrak{S}}^{\mathcal{D}}_{\widetilde{\mathscr{B}}}}(\hat{\ell}_i)$ *is 0.2746 so the laptop* $\hat{\ell}_7$ *is selected.*

## 5. CONCLUSION

In this study, the author has integrated two important concepts: intuitionistic fuzzy parameterization and hypersoft set, to develop a novel theoretical framework called intuitionistic fuzzy parameterized hypersoft set (IFpHSS) which is more adaptable to control information-based vagueness and ambiguities. The definition of IFpHSS, its properties and aggregation operations are explained with illustrated numerical examples. Based on novel notions of IFpHSS, i.e., intuitionistic fuzzy decision set and reduced fuzzy set, a decisive support system is developed with the proposal of an intelligent algorithm. The proposed algorithm is implemented in evaluating a suitable location for the establishment of new educational institution.

1.00,0.00,0.00Future research may explore broader applications of IFpHSS in diverse domains such as medical diagnosis, supply chain management, environmental assessment, and financial analysis, where multi-level uncertain data is prevalent. Additionally, integrating the IFpHSS framework with other computational intelligence techniques such as machine learning, optimization algorithms, or neural networks, could enhance the model's adaptability and decision-making power in dynamic environments. The presented work can further be extended using the ideas presented by Rahman et al. [26, 27].

However, some limitations should be acknowledged. The model's complexity increases with the number of parameters and sub-parameters, which may affect computational efficiency in large-scale problems. Moreover, the current study assumes expert-provided intuitionistic fuzzy values, which may introduce subjectivity. Future efforts could address these challenges by developing automated or data-driven methods for parameter estimation, improving scalability, and conducting comparative analyzes with existing decision-making frameworks to validate the model's relative performance and practicality.

## CONFLICTS OF INTEREST

The author declare no conflict of interest.

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## REFERENCES

[1] Smarandache, F. (2018). Extension of Soft Set of Hypersoft Set, and then to Plithogenic Hypersoft Set, Neutrosophic Sets and Systems, 22, 168-170.

[2] Molodtsov, D. (1999). Soft Set Theory - First Results. Comput Math with Appl., 37, 19-31.

[3] Yolcu, A., Smarandache, F., & Öztürk, T. Y. (2021). Intuitionistic fuzzy hypersoft sets. Communications faculty of sciences University of Ankara Series A1 mathematics and Statistics, 70(1), 443-455.

[4] Zadeh, L. (1965). Fuzzy sets. Information and control, 8(3), 338-353.

[5] Atanassov, K. (1986). Intuitionistic Fuzzy Sets, Fuzzy Sets and Systems, 20, 87-96.

[6] Maji, P. K., Biswas, R., & Roy, A. R. (2001). Fuzzy soft sets. Journal of Fuzzy Mathematics, 9(3), 589-602.

[7] Maji, P. K., Biswas, R., & Roy, A. R. (2001) Intuitionistic fuzzy soft sets, The Journal of Fuzzy Mathematics, 9(3), 677-692.

[8] Ihsan, M., Saeed, M., & Rahman, A. U. (2024). An intuitionistic fuzzy hypersoft expert set-based robust decision-support framework for human resource management integrated with modified topsis and correlation coefficient. Neural Computing and Applications, 36(3), 1123-1147.

[9] Arshad, M., Saeed, M., Rahman, A. U., Bajri, S. A., Alqahtani, H., & Khalifa, H. (2024). Modeling uncertainties associated with multi-attribute decision-making based evaluation of cooling system using interval-valued complex intuitionistic fuzzy hypersoft settings. AIMS Math, 9, 11396-11422.

[10] Sajid, M., Khan, K. A., Rahman, A. U., Bajri, S. A., Alburaikan, A., & Khalifa, H. A. E. W. (2024). A novel algorithmic multi-attribute decision-making framework for solar panel selection using modified aggregations of cubic intuitionistic fuzzy hypersoft set. Heliyon, 10(17), e36508.

[11] Zulqarnain, R. M., & Siddique, I. (2024). An MCGDM technique for weighted average interaction aggregation operator under interval-valued intuitionistic fuzzy hypersoft set environment. Journal of Computational and Cognitive Engineering, 3(3), 295-306.

[12] Naveed, H., & Ali, S. (2025). Addressing decision-making challenges: similarity measures for interval-valued intuitionistic fuzzy hypersoft sets. Decision Making Advances, 3(1), 175-184.

[13] Rani, M. J., & Periyasamy, S. (2024). Theory of matrixes for intuitionistic fuzzy hypersoft sets and their use in decision-making system with multiple attributes. PatternIQ Mining, 1(2), 65-75. https://doi.org/10.70023/piqm24126

[14] Ali, S., Naveed, H., Siddique, I., & Zulqarnain, R. M. (2024). Extension of interaction geometric aggregation operator for material selection using interval-valued intuitionistic fuzzy hypersoft set. Journal of Operations Intelligence, 2(1), 14-35.

[15] Jafar, M. N., Saeed, M., Saeed, A., Ijaz, A., Ashraf, M., & Jarad, F. (2024). Cosine and cotangent similarity measures for intuitionistic fuzzy hypersoft sets with application in MADM problem. Heliyon, 10(7), e27886.

[16] Naveed, H., & Ali, S. (2025). Addressing decision-making challenges: similarity measures for interval-valued intuitionistic fuzzy hypersoft sets. Decision Making Advances, 3(1), 175-184.

[17] Saeed, M., Saeed, M. H., Khalid, M., & Mekawy, I. (2023). Development of hamming and hausdorff distance metrics for cubic intuitionistic fuzzy hypersoft set in cement storage quality control: Development and evaluation. Plos one, 18(9), e0291817.

[18] Jafar, M. N., & Saeed, M. (2024). Some New Aggregation Operations of Intuitionistic Fuzzy Hypersoft Set. HyperSoft Set Methods in Engineering, 2, 83-95.

[19] Çağman, N., Çitak, F., & Enginoğlu, S. (2010). Fuzzy parameterized fuzzy soft set theory and its applications. Turkish Journal of Fuzzy System, 1(1), 21-35.

[20] Çağman, N., Enginoğlu, S., & Çitak, F. (2011). Fuzzy Soft Set Theory and Its Applications. Iran J. Fuzzy Syst., 8(3), 137-147.

[21] Çağman, N., & Karataş, S. (2013). Intuitionistic fuzzy soft set theory and its decision making. Journal of Intelligent and Fuzzy Systems, 24(4), 829-836.

[22] Deli, I., & Çağman, N. (2015). Intuitionistic fuzzy parameterized soft set theory and its decision making. Applied Soft Computing, 28, 109-113.

[23] Karaaslan, F., Çağman, N., & Yilmaz, S. (2013). IFP-intuitionistic fuzzy soft set theory and its applications. arXiv preprint arXiv:1304.0806.

[24] Çağman, N., & Deli, I. (2012). Products of FP-soft sets and their applications. Hacettepe Journal of Mathematics and Statistics, 41(3), 365-374.

[25] Çağman, N., & Deli, I. (2012). Means of FP-soft sets and their applications. Hacettepe Journal of Mathematics and Statistics, 41(5), 615-625.

[26] Rahman, A. U., Smarandache, F., Saeed, M., & Khan, K. A. (2025). Fuzzy parameterized extensions of hypersoft set embedded with possibility-degree settings: an overview. Neutrosophic Sets and Systems, 87, 713-741.

[27] Rahman, A. U., Smarandache, F., Saeed, M., & Khan, K. A. (2025). Development of Some New Hybrid Structures of Hypersoft Set With Possibility-Degree Settings. In F. Smarandache, M. Jdid, & M. Leyva-Vazquez (Eds.), Neutrosophic and Plithogenic Inventory Models for Applied Mathematics (chap:14, pp. 337-386). IGI Global Scientific Publishing. https://doi.org/10.4018/979-8-3693-3204-7.ch014

[28] Ihsan, M., Saeed, M., Rahman, A. U., Kamacı, H., & Ali, N. (2022). An MADM-based fuzzy parameterized framework for solar panels evaluation in a fuzzy hypersoft expert set environment. AIMS Mathematics, 8(2), 3403-3427.

[29] Ihsan, M., Saeed, M., Rahman, A. U., Khalifa, H. A. E. W., & El-Morsy, S. (2022). An intelligent fuzzy parameterized multi-criteria decision-support system based on intuitionistic fuzzy hypersoft expert set for automobile evaluation. Advances in Mechanical Engineering, 14(7), 16878132221110005.