

# Bayes Estimation of Lindley Distribution Under Entropy Loss Functions and Type II Censoring

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**ABSTRACT:** This study presents Bayesian estimation for the parameters of the Lindley distribution under Type II censoring, employing entropy loss functions and conjugate priors. Markov Chain Monte Carlo (MCMC) methods are used to derive the posterior distributions, facilitating parameter estimation. The use of conjugate priors streamlines the computational process, enhancing the practicality of the Bayesian framework. Simulation results confirm the effectiveness of the Bayesian estimators under entropy loss, particularly in censored data scenarios. The methodology offers a valuable approach for researchers and practitioners working with reliability and lifetime data, demonstrating the efficiency and accuracy of the proposed Bayesian methods.

**Keywords:** Prior, Posterior distribution, Loss function, Right censoring.

## 1. INTRODUCTION

The Lindley distribution has been widely used in reliability and lifetime data analysis due to its flexibility and ability to model various types of failure times. Unlike the exponential distribution, the Lindley distribution can account for different types of hazard rates, making it more suitable for real-world applications. However, in many studies, complete data collection is not always possible. Type II censoring occurs when only a portion of the data, such as the first  $v$  observations out of  $n$ , is available, which poses challenges for accurate parameter estimation.

Bayesian methods offer a powerful alternative to traditional estimation approaches, as they incorporate prior information and allow for a more nuanced treatment of uncertainty. When conjugate priors are used, the computational process is greatly simplified, leading to more efficient parameter estimation. Additionally, the use of entropy loss functions improves the estimation process by accounting for discrepancies in the estimates more effectively than other loss functions. In this study, Bayesian estimation was performed using Markov Chain Monte Carlo (MCMC) methods to estimate the parameters of the Lindley distribution under Type II censoring. The MCMC approach enables sampling from the posterior distributions, facilitating parameter estimation when analytical solutions are difficult to obtain. Through this method, we aim to demonstrate the advantages of Bayesian estimation with entropy loss functions in improving the accuracy and reliability of parameter estimates in censored data scenarios.

## 2. Review of literature

**F. P. A. Coolen 1995** examines Bayesian reliability analysis with informative priors and censoring. The study highlights the challenges of censored observations in reliability problems and emphasizes the role of expert judgment in Bayesian methods due to limited statistical data and experimental possibilities. Coolen proposes utilizing hyperparameters of conjugate prior distributions for parametric lifetime models, extending beyond standard conjugate priors to facilitate easier elicitation of prior knowledge based on past censoring experiences. This approach offers greater flexibility in modelling prior knowledge despite the increased computational complexity, which is mitigated by advanced statistical software. **M.E. Ghitany et.al 2008** studied a one parameter distribution by name Lindley distribution and discussed its statistical properties. The properties studied include: moments, cumulants, characteristic function, failure rate function, mean residual life function, mean deviations, Lorenz curve, stochastic ordering, entropies, asymptotic distribution of the extreme order statistics, distributions of sums, products and ratios, maximum likelihood estimation and simulation schemes. An application to waiting time data at a bank was described. This review highlights Hahn, Martin, and Walker's contributions, underscoring the model's versatility and its potential to advance predictive modelling in hydroclimatology. **Debasis Kundu 2012** article explores Bayesian inference for the Weibull distribution under progressive censoring, focusing on unknown parameters within this framework. Recognizing that a continuous conjugate joint prior for the shape and scale parameters of the Weibull distribution is not available, the study assumes a log-concave prior for the shape parameter and a conjugate prior for the scale parameter given the shape. Due to the lack of closed-form expressions for Bayes estimators

when the shape parameter is unknown, Lindley's approximation is employed for estimation, complemented by Gibbs sampling for credible interval calculations. The article also introduces a methodology for comparing different censoring schemes to identify the optimal Bayesian censoring strategy. Monte Carlo simulations validate the proposed methods, and practical data analysis illustrates their application. **Sanjay Kumar Singh, Umesh Singh, and Vikas Kumar Sharma 2014** focuses on Bayesian estimation and prediction for the generalized Lindley distribution under asymmetric loss functions. The authors develop Bayesian estimation procedures for the generalized Lindley distribution using both squared error and general entropy loss functions with complete sample observations. The study employs both non-informative and informative priors to derive Bayes estimates. Monte Carlo simulations are conducted to compare the performance of these Bayesian estimators against maximum likelihood estimators, evaluating their risks. Additionally, the paper addresses Bayesian prediction, deriving prediction intervals for future observations based on an informative sample, and includes numerical examples using real data. **M. R. Hasan and A. R. Baizid 2016** explores Bayesian estimation for the exponential distribution using a gamma prior and compares it with classical methods. The study evaluates Bayes estimators under various loss functions, including squared error, quadratic, modified linear exponential (MLINEX), and non-linear exponential (NLINEX) loss functions. By employing simulated data, the paper assesses the mean squared error (MSE) of these estimators and demonstrates that Bayesian estimators often outperform classical maximum likelihood estimators (MLE) in terms of MSE. The results are illustrated graphically, highlighting the advantages of Bayesian methods under different loss functions. **Farouk Metiri, Halim Zeghdoudi, and Mohamed Riad Remita 2016** investigates Bayesian estimation of the Lindley distribution using Linex loss functions. The study derives the posterior distributions for the Lindley distribution under both non-informative (Jeffreys prior) and informative (Inverted Gamma prior) approaches. The performance of these Bayesian estimators is evaluated through Monte Carlo simulations, focusing on the mean squared error (MSE). The comparison highlights the effectiveness of different priors in the context of Linex loss functions

### 3. Methodology

#### 3.1. Bayesian analysis for Lindley distribution

Let  $x$  be a random variable follow Lindley distribution with parameter  $\theta$  then the pdf of Lindley distribution is given by

$$f(x) = \frac{\theta^2}{\theta + 1} [1 + x] e^{-\theta x} \quad x > 0, \theta > 0 \quad \dots (1)$$

The joint probability function with  $n$  number of sample is given by

$$\begin{aligned} l(x_i) &= \prod_{i=1}^n \frac{\theta^2}{\theta + 1} [1 + x_i] e^{-\theta x_i} \\ &= \prod_{i=1}^n \frac{\theta^2}{\theta + 1} [1 + x_i] e^{-\theta x_i} \\ &= \frac{\theta^{2n}}{(\theta + 1)^n} e^{-\theta \sum_{i=1}^n x_i} \prod_{i=1}^n [1 + x_i] \end{aligned} \quad \dots (2)$$

In this research the conjugate prior is used which gamma distribution with parameter  $r$ . The prior distribution is given by

$$p(\theta) = \frac{e^{-\theta} \cdot \theta^{r-1}}{\Gamma r} \quad r > 0, \theta > 0$$

The posterior distribution with conjugate prior for Lindley distribution is given by

$$\begin{aligned} p(\theta/x) &\propto \frac{e^{-\theta} \cdot \theta^{r-1}}{\Gamma r} \prod_{i=1}^n \frac{\theta^2}{\theta + 1} [1 + x_i] e^{-\theta x_i} \\ &= \frac{1}{k} \frac{e^{-\theta} \cdot \theta^{r-1}}{\Gamma r} \prod_{i=1}^n \frac{\theta^2}{\theta + 1} [1 + x_i] e^{-\theta x_i} \end{aligned} \quad \dots (3)$$

Where

$$k = \int_0^{\infty} \frac{e^{-\theta} \cdot \theta^{r-1}}{\Gamma r} \prod_{i=1}^n \frac{\theta^2}{\theta + 1} [1 + x_i] e^{-\theta x_i} d\theta$$

$$\begin{aligned}
&= \int_0^{\infty} \frac{e^{-\theta} \cdot \theta^{r-1}}{\Gamma r} \frac{\theta^{2n}}{(\theta+1)^n} e^{-\theta \sum_{i=1}^n x_i} \prod_{i=1}^n [1+x_i] d\theta \\
&= \int_0^{\infty} \frac{\prod_{i=1}^n [1+x_i]}{\Gamma r} \frac{\theta^{2n+r-1}}{(\theta+1)^n} e^{-\theta(1+\sum_{i=1}^n x_i)} d\theta \\
&= \frac{\prod_{i=1}^n [1+x_i]}{\Gamma r} \int_0^{\infty} \frac{\theta^{2n+r-1}}{(\theta+1)^n} e^{-\theta(1+\sum_{i=1}^n x_i)} d\theta \\
(1+\theta)^{-n} &= \sum_{j=0}^{\infty} (-1)^j c(n+j-1, j) (\theta)^j \\
&= \frac{\prod_{i=1}^n [1+x_i]}{\Gamma r} \int_0^{\infty} \sum_{j=0}^{\infty} (-1)^j c(n+j-1, j) (\theta)^j \cdot \theta^{2n+r-1} e^{-\theta(1+\sum_{i=1}^n x_i)} d\theta \\
&= \sum_{j=0}^{\infty} (-1)^j c(n+j-1, j) \frac{\prod_{i=1}^n [1+x_i]}{\Gamma r} \int_0^{\infty} \theta^{2n+r+j-1} e^{-\theta(1+\sum_{i=1}^n x_i)} d\theta \\
k &= \sum_{j=0}^{\infty} (-1)^j c(n+j-1, j) \frac{\prod_{i=1}^n [1+x_i]}{\Gamma r} \left[ \frac{\Gamma(2n+r+j)}{(1+\sum_{i=1}^n x_i)^{2n+r+j}} \right] \quad \dots (4)
\end{aligned}$$

Therefore, the posterior distribution of Lindley distribution with conjugate prior is given by

$$\begin{aligned}
p(\theta/x) &= \frac{\Gamma r (1 + \sum_{i=1}^n x_i)^{2n+r+j}}{\sum_{j=0}^{\infty} (-1)^j c(n+j-1, j) \prod_{i=1}^n [1+x_i] [\Gamma(2n+r+j)]} \frac{e^{-\theta} \cdot \theta^{r-1}}{\Gamma r} \frac{\theta^{2n}}{(\theta+1)^n} e^{-\theta \sum_{i=1}^n x_i} \prod_{i=1}^n [1+x_i] \\
p(\theta/x) &= \frac{(1 + \sum_{i=1}^n x_i)^{2n+r+j}}{\sum_{j=0}^{\infty} (-1)^j c(n+j-1, j) [\Gamma(2n+r+j)]} \cdot \frac{\theta^{2n+r-1}}{(\theta+1)^n} e^{-\theta(1+\sum_{i=1}^n x_i)} \\
\frac{(1 + \sum_{i=1}^n x_i)^{2n+r+j}}{\sum_{j=0}^{\infty} (-1)^j c(n+j-1, j) [\Gamma(2n+r+j)]} &= \rho \\
p(\theta/x) &= \rho \cdot \frac{\theta^{2n+r-1}}{(\theta+1)^n} e^{-\theta(1+\sum_{i=1}^n x_i)} \quad \dots (5)
\end{aligned}$$

Bayes estimation of Lindley distribution is given by

$$\begin{aligned}
E[\theta] &= \int_0^{\infty} \theta \rho \cdot \frac{\theta^{2n+r-1}}{(\theta+1)^n} e^{-\theta(1+\sum_{i=1}^n x_i)} d\theta \\
&= \rho \int_0^{\infty} \frac{\theta^{2n+r}}{(\theta+1)^n} e^{-\theta(1+\sum_{i=1}^n x_i)} d\theta
\end{aligned}$$

There are several type of loss functions. in this research the general entropy loss function and entropy loss function was used. while estimating the parameter under this loss functions estimated values of the parameter differs according to the loss function. This states that the loss function was help for estimating the parameter with more accuracy.

The entropy loss function is given by

$$l(\theta, d) = \left(\frac{d}{\theta}\right)^c - c \log\left(\frac{d}{\theta}\right) - 1 \quad \text{where } c \neq 0$$

$\theta$  is the unknown parameter,  $d$  is the decision under consideration for  $\theta$  and  $c$  is the constant. entropy loss function was a special case of general entropy loss function therefore it can be obtained by giving the 1 to constant. Hence the general entropy loss function is given by

$$l(\theta, d) = \left(\frac{d}{\theta}\right) - c \log\left(\frac{d}{\theta}\right) - 1$$

Estimation of posterior distribution under ELF

$$E[l(\theta, d, c)] = \int_0^\infty \left\{ \left( \frac{d}{\theta} \right)^c - c \log \left( \frac{d}{\theta} \right) - 1 \right\} \frac{(1 + \sum_{i=1}^n x_i)^{2n+r+j}}{\sum_{j=0}^\infty (-1)^j c(n+j-1, j) [\Gamma(2n+r+j)]} \cdot \frac{\theta^{2n+r-1}}{(\theta+1)^n} e^{-\theta(1+\sum_{i=1}^n x_i)} d\theta$$

Bayes estimation after estimating with entropy loss function is given

$$\frac{\partial E[l(\theta, d)]}{\partial d} = 0$$

### 3.2. Type -II right censored

The data follows a Lindley distribution with parameter  $\theta$ . Some of the observations are subject to right censoring, meaning that know that the true value is greater than or equal to a certain threshold for these censored data points. the presence of right censoring affects the way which compute the likelihood function for the data. Right censoring occurs when the true value of an observation is only known to be greater than a certain threshold, but the exact value is not observed. To accommodate this in the likelihood, censored and uncensored data points differently.

Let:

- $n$  represent the total number of observations.
- $v$  represents the number of uncensored observations.
- $x_1, x_2, \dots, x_v$  be the uncensored data points.
- $x_v$  represent the largest censored observation (i.e., the threshold above which the remaining observations fall).

The joint pdf of  $x = (x_1, x_2, \dots, x_r)$  under right censoring is given by

$$L_c(x; n, v) = C \prod_{i=1}^v f(x_i) [1 - F(x_r)]^{n-v}$$

$$C = \frac{n!}{(n-v)!}$$

$$F(x) = 1 - \frac{\theta + 1 + \theta x}{\theta + 1} e^{-\theta x}$$

$$L_c(x; n, r) = \frac{n!}{(n-v)!} \prod_{i=1}^v \left( \frac{\theta^2}{\theta + 1} [1 + x_i] e^{-\theta x_i} \right) \left[ \frac{\theta + 1 + \theta x_v}{\theta + 1} e^{-\theta x_v} \right]^{n-v}$$

The posterior distribution was derived by censoring the pdf with right censoring. The posterior distribution with right censoring is given by

$$p_c(\theta/x) = \frac{1}{k} L(x; n, r) \cdot p(\theta) \\ = \frac{1}{k} \frac{n!}{(n-v)!} \prod_{i=1}^v \left( \frac{\theta^2}{\theta + 1} [1 + x_i] e^{-\theta x_i} \right) \left[ \frac{\theta + 1 + \theta x_v}{\theta + 1} e^{-\theta x_r} \right]^{n-v} \cdot \frac{e^{-\theta} \cdot \theta^{r-1}}{\Gamma r}$$

$$\text{Where } k = \int_0^\infty \frac{n!}{(n-v)!} \prod_{i=1}^v \left( \frac{\theta^2}{\theta + 1} [1 + x_i] e^{-\theta x_i} \right) \left[ \frac{\theta + 1 + \theta x_v}{\theta + 1} e^{-\theta x_r} \right]^{n-v} \cdot \frac{e^{-\theta} \cdot \theta^{r-1}}{\Gamma r} d\theta$$

Posterior mean of Lindley distribution with right censoring is given by

$$E_c[\theta] = \int_0^\infty \theta \frac{1}{k} \frac{n!}{(n-v)!} \prod_{i=1}^v \left( \frac{\theta^2}{\theta + 1} [1 + x_i] e^{-\theta x_i} \right) \left[ \frac{\theta + 1 + \theta x_v}{\theta + 1} e^{-\theta x_r} \right]^{n-v} \cdot \frac{e^{-\theta} \cdot \theta^{r-1}}{\Gamma r} d\theta \\ = \int_0^\infty \theta \frac{1}{k} L_c(x; n, r) \cdot p(\theta) d\theta$$

The estimation of posterior distribution with Elf under right censoring is given by

$$E_c[l(\theta, d, c)] = \int_0^{\infty} \left( \frac{d}{\theta} \right)^c - c \log \left( \frac{d}{\theta} \right) - 1 \frac{1}{k} \frac{n!}{(n-v)!} \prod_{i=1}^v \left( \frac{\theta^2}{\theta+1} [1+x_i] e^{-\theta x_i} \right) \left[ \frac{\theta+1+\theta x_v}{\theta+1} e^{-\theta x_r} \right]^{n-v} \cdot \frac{e^{-\theta} \cdot \theta^{r-1}}{\Gamma r} d\theta$$

### 3.3. Simulation study

Simulated data is used with various sample size. `set.seed(123)` is used to initialize the random number generator, ensuring that the sequence of random numbers generated remains consistent and reproducible each time the code is run. This is crucial for replicating results in simulations and statistical analyses. The number 123 is arbitrary and can be any integer, serving simply as a starting point for generating pseudorandom numbers.

The estimation of unknown parameter is highly complicated so the posterior mean (Bayes estimate) of parameter  $\theta$  can be done by using R software. The algorithm for posterior mean is given below.

#### Algorithm Bayesian Estimation Using Metropolis-Hastings (MCMC) Under Right-Censoring

##### Step 1: Likelihood Function

- **Input:** Parameter  $\theta$ , observed data  $x$ , total number of observations  $n$ , and the number of uncensored observations  $v$ .
- **Computation:**
  1. Calculate the likelihood for uncensored data points  $x_1, x_2, \dots, x_n$
  2. Calculate the likelihood for censored data points  $x_{v+1}, x_{v+2}, \dots, x_n$
  3. Multiply these values to obtain the total likelihood.
- **Output:** Likelihood value for given  $\theta$ .

##### Step 2: Prior Distribution

- **Input:** Parameter  $\theta$ , shape parameter  $r$  of the prior Gamma distribution.
- **Computation:**
  1. Use the Gamma distribution  $\text{Gamma}(r, 1)$  to compute the prior probability of  $\theta$ .
- **Output:** Prior probability for  $\theta$ .

##### Step 3: Posterior Distribution

- **Input:** Parameter  $\theta$ , data  $x$ , total number of observations  $n$ , uncensored data count  $v$ , and prior shape parameter  $r$ .
- **Computation:**
  1. Combine the likelihood and prior to compute the posterior probability.
  2. Compute the posterior probability  $P(\theta|x) \propto \text{Likelihood}(\theta) \times \text{Prior}(\theta)$
- **Output:** Posterior probability for  $\theta$ .

##### Step 4: Metropolis-Hastings MCMC Algorithm

- **Initialization:**
  1. Set an initial value for  $\theta$  (e.g. 0).
  2. Set the number of and proposal step size  $\sigma$ .
- **Iteration:**
  1. Propose a new value for  $\theta$  from a normal distribution  $N(\theta, \sigma)$ .
  2. Ensure the proposed value  $\theta_{\text{proposed}}$  is positive.
  3. Compute the posterior probabilities for both  $\theta_{\text{current}}$  and  $\theta_{\text{proposed}}$ .
  4. Calculate the acceptance ratio  $\alpha = \frac{P(\theta_{\text{current}}|x)}{P(\theta_{\text{proposed}}|x)}$
  5. Accept  $\theta_{\text{proposed}}$  with probability  $\min(1, \alpha)$ ; otherwise, retain  $\theta_{\text{current}}$ .
- **Repeat:**
  1. Repeat the process for all iterations.
  2. Store the accepted values of  $\theta$ .

##### Step 5: Parameter Estimation and Loss Function

- **Estimate  $\theta$ :** Compute the posterior mean and median of  $\theta$  from the samples generated by the MCMC algorithm.
- **Entropy Loss Minimization:**
  1. Define a range of potential values for decision parameter  $d$ .
  2. For each  $d$ , compute the expected entropy loss using the sampled  $\theta$  values.
  3. Identify the optimal  $d$  that minimizes the expected entropy loss.

##### Step 6: Visualization

- **Trace Plot:** Plot the trace of  $\theta$  values over iterations to check for convergence.

- **Posterior Distribution:** Plot the histogram of the posterior distribution and mark the posterior mean.

Table :1 Posterior mean (without loss function)

| N   | $E[\theta]$ |
|-----|-------------|
| 10  | 7.051       |
| 50  | 12.11       |
| 100 | 49.18       |

Table :2 Posterior mean with entropy loss function and Minimizing Expected Entropy Loss

| n   | $E[\theta_{ELF}]$ | Minimizing Expected Entropy Loss |
|-----|-------------------|----------------------------------|
| 10  | 4.226559          | 3.8                              |
| 50  | 5.625803          | 5.4                              |
| 100 | 5.848879          | 5.7                              |

Table :3 Posterior mean with General entropy loss function and Minimizing Expected Entropy Loss

| n   | $E[\theta_{GELF}]$ | Minimizing Expected General Entropy Loss |
|-----|--------------------|------------------------------------------|
| 10  | 4.226559           | 4                                        |
| 50  | 5.625803           | 5.5                                      |
| 100 | 5.848879           | 5.8                                      |

Table :4 Posterior mean under right censoring  $V = 70\%$  of the total sample size and  $r = 2$ 

| n   | $E_c[\theta]$ |
|-----|---------------|
| 10  | 4.03021       |
| 50  | 2.537439      |
| 100 | 2.679314      |

### 3.4.Real life dataset

Posterior mean for lindley distribution with vaiour loss function and under right censoring.

Table 5. shows the life of fatigue fracture of Kevlar 373/epoxy subjected to constant pressure at 90 % stress level until all had failed.

|                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                              |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| 0.0251, 0.0886, 0.0891 0.2501, 0.3113, 0.3451, 0.4763, 0.5650, 0.5671, 0.6566, 0.6748, 0.6751, 0.6753, 0.7696, 0.8375, 0.8391, 0.8425, 0.8645, 0.8851, 0.9113, 0.9120, 0.9836, 1.0483, 1.0596, 1.0773, 1.1733, 1.2570, 1.2766, 1.2985, 1.3211, 1.3503, 1.3551, 1.4595, 1.4880, 1.5728, 1.5733, 1.7083, 1.7263, 1.7460, 1.7630, 1.7746, 1.8475, 1.8375, 1.8503, 1.8808, 1.8878, 1.8881, 1.9316, 1.9558, 2.0048, 2.0408, 2.0903, 2.1093, 2.1330, 2.2100, 2.2460, 2.2878, 2.3203, 2.3470, 2.3513, 2.4951, 2.5260, 2.9911, 3.0256, 3.2678, 3.4045, 3.4846, 3.7433, 3.7455, 3.9143, 4.8073, 5.4005, 5.4435, 5.5295, 6.5541, 9.09. |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|

Table :6 The posterior mean of real-life data

|                      | Posterior Mean |
|----------------------|----------------|
| $E[\theta]$          | 0.8027839      |
| $E[\theta_{ELF}]$    | 0.7233663      |
| $E[\theta_{GELF}]$   | 0.7927012      |
| $E_c[\theta]$        | 0.9415085      |
| $E_c[\theta_{ELF}]$  | 0.9418115      |
| $E_c[\theta_{GELF}]$ | 0.9432054      |

#### 4. Discussion

In this study, the Metropolis-Hastings (MH) algorithm was applied to estimate the parameter  $\theta$  for data that includes right-censoring. This method allowed us to incorporate prior information and effectively address the complexities of censored data. Our results were analysed across different scenarios and with various loss functions.

##### 4.1. Simulation Study Results

The estimation procedure using simulated data with different sample sizes( $n$ ) such as 10,50 and 100. The posterior mean are 7.051, 12.11 and 49.18. These results show that the estimated value of  $\theta$  increases as the sample size grows, indicating that larger samples provide more precise estimates.

##### 4.2. Analysis Using Different Loss Functions

The posterior mean values for different sample sizes are as follows: for  $n=10$ , the posterior mean is 4.226559, for  $n=50$ , it is 5.625803, and for  $n=100$ , the value is 5.848879. When minimizing expected entropy loss (ELF), the results are slightly lower: for  $n=10$ , the ELF is 3.8, for  $n=50$ , it is 5.4, and for  $n=100$ , it reaches 5.7. Similarly, minimizing general entropy loss (GELF) yields values closer to the posterior mean: for  $n=10$ , the GELF is 4.0, for  $n=50$ , it is 5.5, and for  $n=100$ , it reaches 5.8. The variations in the posterior mean across different loss functions highlight how the choice of loss function can affect the estimation of  $\theta$ .

##### 4.3. Right-Censoring Analysis

For data with 70% right-censoring and using a Gamma prior with a shape parameter of  $r=2$ , the posterior means vary with the sample size. For a sample size of  $n=10$ , the posterior mean is 4.03021. As the sample size increases, the posterior mean decreases: for  $n=50$ , it is 2.537439, and for  $n=100$ , the value is 2.679314. These values show that the estimated  $\theta$  tends to decrease as the sample size increases, which might be related to the higher proportion of censored data.

##### 4.4. Real-Life Dataset Analysis

For the real-life dataset provided and compared the posterior means with and without different loss functions. The posterior mean without any loss function is 0.8027839, while incorporating loss functions changes the results slightly. When using the expected entropy loss (ELF), the posterior mean is reduced to 0.7233663, and with the general entropy loss (GELF), it is 0.7927012. For the right-censoring data, without any loss function, the value is 0.9415085. However, with ELF, the right-censoring value slightly increases to 0.9418115, with a mean loss of 2.100451. When applying GELF, the right-censoring value becomes 0.9432054, with a much lower mean loss of 0.3806907. This shows the impact of different loss functions on both the posterior mean and the performance under right-censoring conditions. The results show that applying loss functions changes the posterior mean slightly. GELF, in particular, resulted in a lower mean loss compared to ELF, suggesting it might be more effective in this context.

#### 5. Conclusion

The Metropolis-Hastings algorithm provided a robust method for estimating  $\theta$ , even with right-censored data. The results indicate that the posterior mean of  $\theta$  varies with sample size, loss functions, and the proportion of censored data. Choosing the appropriate loss function is crucial for accurate estimation, with GELF

demonstrating better performance in terms of minimizing loss. Future research could explore other loss functions or priors to further improve the estimation process for censored data.

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