

# On Neutrosophic Relations in Group Theory

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**ABSTRACT:** In this paper, we study neutrosophic of some triple relations or neutrosophic triple set (NTS) and related to the homomorphism groups. Various properties like equivalent relation, product, intersection, and the union are studied. We showed several equivalent relations in group theory with new examples about neutrosophic homomorphism. Finally, some definitions, examples, and other remarks of neutrosophic of these relations are given.

**Keywords:** Neutrosophic group, Neutrosophic set, Multiplication module, Relation, Homomorphism group.

## 1. INTRODUCTION

In 1980, Smarandache introduced new notion in mathematics is called neutrosophic theory. In neutrosophic, we consider the main principal which is named logic where approach to the truth T, indeterminacy I, and the falsity F. Indeterminacy deals with fuzzy theory and others theories. All facts about fuzzy set in [1]. W. B. Vasantha Kandasamy and Florentin Smarandache introduced more concepts like neutrosophic group, neutrosophic rings and neutrosophic semigroups. In [2] and [3], to find more results and information about neutrosophic relations. In [4], shwed some new results of neutrosophic multiplication module with some properties of neutrosophic set. Salama and Alblowi, 2012 [5], studied Neutrosophic set and neutrosophic topological space. Neutrosophic Groups and Subgroups in [6]. Soft neutrosophic semigroup and their generalization in [7]. Single valued neutrosophic mappings defined by single valued neutrosophic relations with applications in [8]. Neutrosophic BCK-algebra and  $\Omega$ -BCK-algebra [9]. Some results of neutrosophic topological rings [10]. More details about the relations and some applications in [11-14] This paper is about to introduced neutrosophic of relations in group theory like homomorphism and equivalent relations such as reflexive, symmetric and transitive.

## 2. PRELIMINARIES

This section is about some of the various definitions, remarks and some basic properties which are used later in this notion. Also, we recall more facts related to neutrosophic sets [4, main paper].

**Definition 2.1.** [15] Let  $X \neq \emptyset$  be a set. Any set which is generated by  $X$  and  $I$  is called neutrosophic and denoted by  $X(I)$ . Or let  $S$  be a collection of points with an element in  $S$  like  $b$ . A neutrosophic set  $B$  in  $S$  is truth function  $T_B$ , Indeterminacy function  $I_B$  and falsity function  $F_B$ .

$T_B(a)$ ,  $I_B(a)$  and  $F_B(a)$  are real and subsets of  $[0, 1]$  which written as:

$B = \{ \langle b, (T_B(b), I_B(b), F_B(b)) \rangle : b \in \Omega, T(b), I(b), F(b) \in [0, 1] \}$ .

**Remark 2.2.** We can say:

$0 \leq T_B(b) + I_B(b) + F_B(b) \leq 3$ .

**Definition 2.3.** Suppose that  $\Omega$  be a universe. We define a neutrosophic triple set NTS( $B$ ) on  $\Omega$  by the following:

$B = \{ \langle b, (T_B^{1,2,3}(b), I_B^{1,2,3}(b), F_B^{1,2,3}(b)) \rangle : b \in \Omega \}$  where  $T_B^{1,2,3}(b) : \Omega \rightarrow [0, 1]$ ,  $I_B^{1,2,3}(b) : \Omega \rightarrow [0, 1]$ ,  $F_B^{1,2,3}(b) : \Omega \rightarrow [0, 1]$ .

**Remark 2.4.**

1)  $0 \leq T_B^i(b) + I_B^i(b) + F_B^i(b) \exists i=1, 2, 3$  and  $T_B^1(b) \leq T_B^2(b) \leq T_B^3(b)$ ,  $b \in \Omega$ .

2) NTS( $\Omega$ ) refers to the set of all neutrosophic triple sets on  $\Omega$ .

3) a) Assume that  $B$  and  $C$  belong to neutrosophic triple set of  $\Omega$ . Then, the following statements are holds:

a) If  $T_B^i(b)=1$  and  $I_B^i(b)=0=F_B^i(b)$ ,  $b \in \Omega$ , so  $B$  named universal neutrosophic set  $\Omega^*$ .

b)  $B$  is neutrosophic subset of  $C$  ( $B \subseteq C$ ) when:

$T_B^i(b) \leq T_C^i(b)$ ,  $I_B^i(b) \geq I_C^i(b)$  and  $F_B^i(b) \geq F_C^i(b)$ ,  $b \in \Omega$ .

**Definition 2.5.**  $B^*$  is complement of  $B$  such that:

$B^* = \{ \langle b, (F_B^{1,2,3}(b), I_B^{1,2,3}(b), T_B^{1,2,3}(b)) \rangle : b \in \Omega \}$ .

### 3. OPERATIONS OF NST( $\Omega$ )

In this section, we present more properties of neutrosophic set with several examples.

#### Remark 3.1.

a) Assume that B and C belong to neutrosophic triple set of  $\Omega$ . So,

$$B \cup C = \{ \langle b, (T_B \cup C^{1,2,3}(b), I_B \cup C^{1,2,3}(b), F_B \cup C^{1,2,3}(b)) \rangle : b \in \Omega \}.$$

$$T_B^i(b) \vee T_C^i(b) = T_B \cup C^i(b)$$

$$I_B^i(b) \vee I_C^i(b) = I_B \cup C^i(b)$$

$$F_B^i(b) \vee F_C^i(b) = F_B \cup C^i(b)$$

$$b) B \cap C = \{ \langle b, (T_{B \cap C}^{1,2,3}(b), I_{B \cap C}^{1,2,3}(b), F_{B \cap C}^{1,2,3}(b)) \rangle : b \in \Omega \}.$$

$$T_B^i(b) \wedge T_C^i(b) = T_{B \cap C}^i(b)$$

$$I_B^i(b) \wedge I_C^i(b) = I_{B \cap C}^i(b)$$

$$F_B^i(b) \wedge F_C^i(b) = F_{B \cap C}^i(b)$$

$$c) B + C = \{ \langle b, (T_{B+C}^{1,2,3}(b), I_{B+C}^{1,2,3}(b), F_{B+C}^{1,2,3}(b)) \rangle : b \in \Omega \}.$$

$$T_B^i(b) + T_C^i(b) - T_B^i(b) \cdot T_C^i(b) = T_{B+C}^i(b)$$

$$I_B^i(b) + I_C^i(b) - I_B^i(b) \cdot I_C^i(b) = I_{B+C}^i(b)$$

$$F_B^i(b) + F_C^i(b) - F_B^i(b) \cdot F_C^i(b) = F_{B+C}^i(b), b \in \Omega, i=1, 2, 3.$$

$$d) B \times C = \{ \langle b, (T_{B \times C}^{1,2,3}(b), I_{B \times C}^{1,2,3}(b), F_{B \times C}^{1,2,3}(b)) \rangle : b \in \Omega \}.$$

#### Remark 3.2.

$$T_{B \times C}^i(b) = T_B^i(b) \cdot T_C^i(b)$$

$$I_{B \times C}^i(b) + I_{B \times C}^i(b) - I_{B \times C}^i(b) \cdot I_{B \times C}^i(b) = I_{B \times C}^i(b)$$

$$F_{B \times C}^i(b) = F_B^i(b) \cdot F_C^i(b), b \in \Omega, i=1, 2, 3.$$

Now we study some types of neutrosophic relations. We define and describe all properties of  $B \times C$  as a cartesian product.

#### Definition 3.3.

Suppose that we have two non-zero of NTS( $\Omega$ ) with  $i=1, 2, 3$ . So the cartesian product of B and C is named neutrosophic triple sets inside  $\Omega \times \Omega$ :

$$B \times C = \{ \langle (b, c), (T_{B \times C}^i(b, c), I_{B \times C}^i(b, c), F_{B \times C}^i(b, c)) \rangle : (b, c) \in \Omega \times \Omega \}$$

where

$$T_{B \times C}^i(b, c) = \min \{ T_B^i(b, c), T_C^i(b, c) \}$$

$$I_{B \times C}^i(b, c) = \max \{ I_B^i(b, c), I_C^i(b, c) \}$$

$$F_{B \times C}^i(b, c) = \max \{ F_B^i(b, c), F_C^i(b, c) \}, b, c \in B \times C, i=1, 2, 3.$$

#### Remark 3.4.

$$B \times C = \{ \langle (b, c), (T_{B \times C}^i(b, c), I_{B \times C}^i(b, c), F_{B \times C}^i(b, c)) \rangle : (b, c) \in \Omega \times \Omega \}, i=1, 2, 3 \text{ with}$$

$$T_{B \times C}^i : \Omega \times \Omega \rightarrow [0, 1].$$

Note that we define a triple product of neutrosophic triple sets by the following:

#### Definition 3.5.

Let B, C and D are non-zero sets of neutrosophic triple sets:  $i \in \{1, 2, 3\}$ . The neutrosophic triple direct product of B, C and D in  $\Omega \times \Omega \times \Omega$ :

$$B \times C \times D = \{ \langle (b, c, d), (T_{B \times C \times D}^{1,2,3}(b, c, d), I_{B \times C \times D}^{1,2,3}(b, c, d), F_{B \times C \times D}^{1,2,3}(b, c, d)) \rangle : (b, c, d) \in \Omega \times \Omega \times \Omega \}, i=1, 2, 3 \text{ with}$$

$$T_{B \times C \times D}^i(b, c, d) = \min \{ T_B^i(b, c, d), T_C^i(b, c, d), T_D^i(b, c, d) \}$$

$$I_{B \times C \times D}^i(b, c, d) = \max \{ I_B^i(b, c, d), I_C^i(b, c, d), I_D^i(b, c, d) \}$$

$$F_{B \times C \times D}^i(b, c, d) = \max \{ F_B^i(b, c, d), F_C^i(b, c, d), F_D^i(b, c, d) \}, b, c, d \in B \times C, i=1, 2, 3.$$

#### Definition 3.6.

Let B, C, D  $\in$  NTS( $\Omega$ ) and let  $R_1$ ,  $R_2$  and  $R_3$  be three neutrosophic triple relations from  $(B \times C) \rightarrow D$ . Then we define  $(R_1 \cup R_2) \cup R_3$ ,  $(R_1 \cap R_2) \cap R_3$ ,  $(R_1 + R_2) + R_3$  and  $(R_1 \times R_2) \times R_3$  by the following:

$$(R_1 \cup R_2) \cup R_3 = \{ \langle ((b, c), d), T_{(R_1 \cup R_2) \cup R_3}^{1,2,3}((b, c), d), I_{(R_1 \cup R_2) \cup R_3}^{1,2,3}((b, c), d), F_{(R_1 \cup R_2) \cup R_3}^{1,2,3}((b, c), d) \rangle : ((b, c), d) \in \Omega \}.$$

#### Remark 3.7.

$$T_{(R_1 \cup R_2) \cup R_3}^i((b, c), d) = T_{R_1 \cup R_2}^i((b, c), d) \vee T_{R_3}^i((b, c), d),$$

$$I_{(R_1 \cup R_2) \cup R_3}^i((b, c), d) = I_{R_1 \cup R_2}^i((b, c), d) \wedge T_{R_3}^i((b, c), d),$$

$$F_{(R_1 \cup R_2) \cup R_3}^i((b, c), d) = F_{R_1 \cup R_2}^i((b, c), d) \wedge F_{R_3}^i((b, c), d), \text{ for all } (b, c), d \in \Omega, i=1, 2, 3.$$

$$(R_1 \cap R_2) \cap R_3 = \{ \langle ((b, c), d), T_{(R_1 \cap R_2) \cap R_3}^{1,2,3}((b, c), d), I_{(R_1 \cap R_2) \cap R_3}^{1,2,3}((b, c), d), F_{(R_1 \cap R_2) \cap R_3}^{1,2,3}((b, c), d) \rangle : ((b, c), d) \in \Omega \}.$$

$$F_{(R_1 \cap R_2) \cap R_3}^i((b, c), d) = F_{R_1 \cap R_2}^i((b, c), d) \wedge F_{R_3}^i((b, c), d), \text{ for all } (b, c), d \in \Omega, i=1, 2, 3.$$

#### Remark 3.8.

$$\begin{aligned}
T^i_{(R_1 \cap R_2) \cap R_3}((b, c), d) &= T^i_{R_1 \cap R_2}(b, c) \wedge T^i_{R_3}(c), \\
I^i_{(R_1 \cap R_2) \cap R_3}((b, c), d) &= I^i_{R_1 \cap R_2}(b, c) \vee I^i_{R_3}(c), \\
F^i_{(R_1 \cap R_2) \cap R_3}((b, c), d) &= I^i_{R_1 \cap R_2}(b, c) \vee F^i_{R_3}(c), \text{ for all } (b, c), d \text{ is belonging to } \Omega, i=1, 2, 3. \\
(R_1+R_2)+R_3 &= \{<(b, c), d, T^{1, 2, 3}_{(R_1+R_2)+R_3}((b, c), d), I^{1, 2, 3}_{(R_1+R_2)+R_3}((b, c), d), F^{1, 2, 3}_{(R_1+R_2)+R_3}((b, c), d) >: ((b, c), d) \in \Omega\}.
\end{aligned}$$

**Remark 3.9.**

$$\begin{aligned}
T^i_{(R_1+R_2)+R_3}((b, c), d) &= T^i_{R_1+R_2}(b, c) \wedge T^i_{R_3}(c), \\
I^i_{(R_1+R_2)+R_3}((b, c), d) &= I^i_{R_1+R_2}(b, c) \wedge I^i_{R_3}(c), \\
F^i_{(R_1+R_2)+R_3}((b, c), d) &= F^i_{R_1+R_2}(b, c) \wedge F^i_{R_3}(c), \text{ for all } (b, c), d \text{ is belonging to } \Omega, i=1, 2, 3.
\end{aligned}$$

$$(R_1 \times R_2) \times R_3 = \{<(b, c), d, T^{1, 2, 3}_{(R_1 \times R_2) \times R_3}((b, c), d), I^{1, 2, 3}_{(R_1 \times R_2) \times R_3}((b, c), d), F^{1, 2, 3}_{(R_1 \times R_2) \times R_3}((b, c), d) >: ((b, c), d) \in \Omega\}.$$

**Remark 3.10.**

$$\begin{aligned}
T^i_{(R_1 \times R_2) \times R_3}((b, c), d) &= T^i_{R_1 \times R_2}(b, c) \wedge T^i_{R_3}(c), \\
I^i_{(R_1 \times R_2) \times R_3}((b, c), d) &= I^i_{R_1 \times R_2}(b, c) \wedge I^i_{R_3}(c), \\
F^i_{(R_1 \times R_2) \times R_3}((b, c), d) &= F^i_{R_1 \times R_2}(b, c) \wedge F^i_{R_3}(c), \text{ for all } (b, c), d \text{ is belonging to } \Omega, i=1, 2, 3.
\end{aligned}$$

Note that if  $\phi=S_1, S_2, S_3$  in neutrosophic triple sets of  $\Omega$ , so any two neutrosophic triple sets under composition. Suppose that  $R_1(S_1 \rightarrow S_2)$  and  $R_2(S_2 \rightarrow S_3)$  is a two neutrosophic, we can define  $R_2 \circ R_1$  and it is neutrosophic triple relation from  $S_1$  to  $S_3$ :

$$R_2 \circ R_1 = \{<(a, c), (T^{1, 2, 3}_{R_2 \circ R_1}(a, c), (I^{1, 2, 3}_{R_2 \circ R_1}(a, c), I^{1, 2, 3}_{R_2 \circ R_1}(a, c), (F^{1, 2, 3}_{R_2 \circ R_1}(a, c))) >: a, c \in \Omega\}.$$

**Remark 3.11.**

$$\begin{aligned}
T^i_{R_2 \circ R_1}(a, c) &= V_y \{T^i_{R_1}(a, b) \wedge T^i_{R_2}(b, c)\} \\
I^i_{R_2 \circ R_1}(a, c) &= V_y \{I^i_{R_1}(a, b) \vee I^i_{R_2}(b, c)\} \\
F^i_{R_2 \circ R_1}(a, c) &= V_y \{F^i_{R_1}(a, b) \vee F^i_{R_2}(b, c)\} \text{ for all } (a, c) \in \Omega \times \Omega, y \in \Omega, i=1, 2, 3.
\end{aligned}$$

**Neutrosophic equivalent relation:**

1.  $T^i_{R_1}(a, a) = 1, I^i_{R_1}(a, a) = 0$  and  $F^i_{R_1}(a, a) = 0$ ,  $a$  belongs to  $\Omega$ , then this relation is reflexive.
2. If  $T^i_{R_1}(a, b) = T^i_{R_1}(b, a), I^i_{R_1}(a, b) = I^i_{R_1}(b, a)$  and  $F^i_{R_1}(a, b) = F^i_{R_1}(b, a) \forall a, b \in \Omega$ , then this relation  $R$  is symmetric.
3. If  $R \circ R \subseteq R$ , then  $R$  is transitive.
4. If  $R$  satisfies (1), (2) and (3), then  $R$  is called neutrosophic triple equivalence.
5. If  $R = R \cup R_2 \cup R_3$ , then  $R$  is transitive closure of neutrosophic triple relation.

**Definition 3.12.**

Let  $R$  be a neutrosophic triple relation ( $R: S_1 \rightarrow S_2$ ). So, the inverse neutrosophic triple relation (INT) is  $R^{-1}$ :

$$R^{-1} = \{<(a, b), (T^i_{R^{-1}}(a, b), I^i_{R^{-1}}(a, b), F^i_{R^{-1}}(a, b)) >: (a, b) \in \Omega \times \Omega\}.$$

**Remark 3.13.**

$$\begin{aligned}
T^i_{R^{-1}}(a, b) &= T^i_R(b, a) \\
I^i_{R^{-1}}(a, b) &= I^i_R(b, a) \\
F^i_{R^{-1}}(a, b) &= F^i_R(b, a), j=1, 2, 3.
\end{aligned}$$

#### 4. NEUTROSOPHIC DOMAIN AND THE CODOMAIN (NCOD)b

Let  $S_1$  and  $S_2$  be two sets with neutrosophic triple relation  $R$  has order pair  $(a, b)$ . So the neutrosophic domain of  $R$  (NDR) is:

$$NDR = \{<a : (a, b), T^{1, 2, 3}_D(a, b), I^{1, 2, 3}_D(a, b), F^{1, 2, 3}_D(a, b) >: (a, b) \in \Omega\}.$$

Also, we can present neutrosophic of the codomain (CD) by the following:

$$NCDR = \{<a : (b, a), T^{1, 2, 3}_{CD}(b, a), I^{1, 2, 3}_{CD}(b, a), F^{1, 2, 3}_{CD}(b, a) >: (b, a) \in \Omega\}.$$

**Example 4.1.**

Let  $S_1 = \{1, 2, 9\}$  and  $S_2 = \{1, 3, 7\}$ :

- 1) If neutrosophic triple relation  $R$  is  $NR = \{(1, 1), (3, 3)\}$ , then  $NDR = \{1, 3\}$  and  $NCODR = \{1, 3\}$ .
- 2) If  $R = \{(1, 3), (1, 7), (2, 3), (2, 7)\}$ , then  $NDR = \{1, 2\}$  and  $NCODR = \{3, 7\}$ .
- 3) If  $R = \{(2, 1), (9, 1), (9, 3), (9, 7)\}$ , then  $NDR = \{2, 9\}$  and  $NCODR = \{1, 3, 7\}$ .

**Definition 4.2.**

The neutrosophic triple identity relation on  $S$  is  $\{(a, a) : a \in \Omega\}$  and:

$$NID = \{<a : (a, a), T^{1, 2, 3}_{NID}(a, a), I^{1, 2, 3}_{NID}(a, a), F^{1, 2, 3}_{NID}(a, a) >: (a, a) \in \Omega\}.$$

**Example 4.3.**

The neutrosophic triple empty set is the subset itself. It is clearly neutrosophic triple irreflexive not neutrosophic triple reflexive.

**Example 4.4.**

Consider the neutrosophic triple relation  $R$  on  $S=\{1, 2, 3, 4\}$  and defined by:

$R=\{(1, 1), (2, 3), (2, 4), (3, 3), (3, 4)\}$ . Note that  $(2, 2) \notin R$  and  $(1, 1) \in R$ , so the neutrosophic triple relation is neither reflexive nor irreflexive. Also, we have  $(2, 3) \in R$  but  $(3, 2) \notin R$ , then  $R$  is not neutrosophic triple relation symmetric. On the other hand, if we have  $a \neq b$ , then only one of the following true:

$(a, b) \notin R, (b, a) \notin R, (a, b) \in R$  or  $(b, a) \in R$ .

## 5. NEUTROSOPHIC HOMOMORPHISM GROUP

**Definition 5.1.**

Let  $(G, *)$  be a group. If  $\langle G \cup I \rangle = \{a+bI : a, b \in G\}$ , then  $(\langle G \cup I \rangle, *)$  is called neutrosophic group, where  $G, I$  are generators of NG.

**Definition 5.2.**

The neutrosophic homomorphism  $f$  (denote by  $Nf$ ) between two groups  $G_1$  and  $G_2$  is:

$Nf: NG_1 \rightarrow NG_2 \ni N(f(a*b)) = N(f(a)*N(f(b)))$

**Definition 5.3.**

The neutrosophic triple homomorphism can define by:

$N(f(a*b)) = \{a: (a*b), (T^{1,2,3}_f(a, b), I^{1,2,3}_f(a, b), F^{1,2,3}_f(a, b)) : a, b \in G\}$  where:

$T^i_f(a, b) = T^i_f(a)*T^i_f(b)$ ,

$I^i_f(a, b) = I^i_f(a)*I^i_f(b)$ ,

$F^i_f(a, b) = F^i_f(a)*F^i_f(b)$ ,  $i=1, 2, 3$ .

**Example 5.4.**

Let  $NG_1 = (\langle G_1 \cup I \rangle)$  and  $NG_2 = (\langle G_2 \cup I \rangle)$  be two neutrosophic groups with neutrosophic identity elements  $Ne_1$  and  $Ne_2$ , respectively. Then

$Nf: (G_1 \cup I) \rightarrow (G_2 \cup I)$  define by:

$$\begin{aligned} N(f(a*b)) &= Ne_2 \\ &= Ne_2 \circ Ne_2 \\ &= N(f(a)) \circ N(f(b)). \end{aligned}$$

Hence the neutrosophic triple identity homomorphism is:

$N(f(a*b)) = T^{1,2,3}_f(a*b)$ :

$T^i_f(a*b) = T^i_f(a) \circ T^i_f(b)$

$= Ne_2 \circ Ne_2$

$= Ne_2$ ,

$I^i_f(a*b) = I^i_f(a) \circ I^i_f(b)$

$= Ne_2 \circ Ne_2$

$= Ne_2$ ,

$F^i_f(a*b) = F^i_f(a) \circ F^i_f(b)$

$= Ne_2 \circ Ne_2$

$= Ne_2$ .

### Some Applications :

**Proposition 5.5.**

Let  $(G_1, +) \rightarrow (G_2 - \{0\}, .)$  be a homomorphism group and let  $f(a) = 2^a$ . Then  $f$  is a neutrosophic triple homomorphism.

**Proof:** Let  $N(f(a)) = 2^a$ . Then

$T^i_f(a+b) = T^{1,2,3}_f(a) \circ T^{1,2,3}_f(b) = 2^{a+b} = 2^a \cdot 2^b$ ,

$I^i_f(a+b) = I^{1,2,3}_f(a) \circ I^{1,2,3}_f(b) = 2^{a+b} = 2^a \cdot 2^b$ , and

$F^i_f(a+b) = F^{1,2,3}_f(a) \circ F^{1,2,3}_f(b) = 2^{a+b} = 2^a \cdot 2^b$ .

Thus, Then  $f$  is a neutrosophic triple homomorphism.

**Proposition 5.6.**

Let  $(Z, +)$  be a group of integer numbers and let  $(Z_n, +_n)$  be the group of integer numbers modulo  $n$ . If  $f: (Z, +) \rightarrow (Z_n, +_n)$  be a homomorphism groups defined by  $f(a) = [a]$ , then  $f$  is neutrosophic triple homomorphism.

**Proof:** Clear that

$T_f(a, b) = \{a, (a, b), T^{1,2,3}_f(a, b), I^{1,2,3}_f(b, a), F^{1,2,3}_f(b, a)\} = (b, a) \in \Omega\}$

$$\begin{aligned}
 [a+b] &= T^{1,2,3} f(a, b) = T^{1,2,3} f(a) + T^{1,2,3} f(b), \\
 [a+b] &= I^{1,2,3} f(a, b) = I^{1,2,3} f(a) + I^{1,2,3} f(b), \\
 [a+b] &= F^{1,2,3} f(a, b) = F^{1,2,3} f(a) + F^{1,2,3} f(b).
 \end{aligned}$$

**Proposition 5.7.**

The pair  $(\text{hom } G, \circ)$  where  $(\circ)$  denotes functional composition, forms a semigroup with identity.

**Proof:**  $f \circ g = \{ \langle (a, b), (T^{1,2,3} f g(a, b), (I^{1,2,3} f g(a, b), (F^{1,2,3} f g(a, b)) \rangle : a, b \in G \}$ , where

$$\begin{aligned}
 \{(T^j f g(a, b))\} &= \{ (T^j f g(a))^* (T^j f g(a)) \}, \\
 \{(I^j f g(a, b))\} &= \{ (I^j f g(a))^* (I^j f g(a)) \}, \\
 \{(F^j f g(a, b))\} &= \{ (F^j f g(a))^* (F^j f g(a)) \}
 \end{aligned}$$

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