Nonlinear Asian Option Pricing: A Power Series Solution Framework

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Received 23.01.2026, Revised 22.04.2026, Accepted 25.05.2026

ABSTRACT: A fundamental challenge in financial mathematics lies in accurately pricing Asian options, especially due to their inherent path dependency. This work introduces a pricing approach for Asian options within a nonlinear Black-Scholes framework utilizing the Power Series Method (PSM). Leveraging the path-dependent nature of Asian options, the PSM approximates the governing nonlinear partial differential equation through a Maclaurin series expansion, enabling recursive computation of coefficients. This method is well-suited for both symbolic and numerical analysis. Numerical experiments demonstrate that the PSM achieves high accuracy and computational efficiency, outperforming traditional methods such as the Explicit Forward Difference Method and Monte Carlo Simulation within specified parameter ranges. These results highlight the PSM's potential as a robust and efficient tool for pricing complex derivatives in nonlinear market models.

Keywords: Asian Option, Power Series,Nonlinear Black-Scholes Equation, Maclaurin Series, Option Pricing.

1. INTRODUCTION

The problem of option pricing requires solving the Black-Scholes equation, which may be linear or non-linear. In many cases, the presence of nonlinearity makes both the analysis and solution of the equation more challenging. As a result, advanced numerical and semi-analytical techniques are often necessary. One such approach is the power series method, which has shown promise as an effective method for approximating solutions to nonlinear contingent claim partial differential equations (PDEs). Nonlinear effects in derivative pricing such as those arising from transaction costs, liquidity constraints, and the market impact of large traders have been a central focus of option pricing research since Wilmott's seminal work [1]. For Asian options, the interaction between path-dependence and market microstructure introduces unique computational challenges requiring specialized nonlinear PDE methods.

Nonlinear Model Development: Risk-minimization frameworks and market impact models [2, 3] extended classical theory, with rigorous existence and uniqueness results for nonlinear Black-Scholes PDEs established through viscosity solution theory [4–6]. Analyses of finite liquidity and stochastic volatility [7–9] deepened understanding of nonlinear pricing dynamics through Hamilton-Jacobi-Bellman (HJB) frameworks that solve optimal control via value functions under jump-diffusion and currency uncertainty.

Numerical Methods for Nonlinear PDEs: Robust finite difference schemes ensuring monotonicity and stability were developed [10–13], with applications to emerging markets [14, 15]. The Power Series Method [16, 17] demonstrated exceptional accuracy for nonlinear problems. Modern implementations integrate GARCH volatility [18], Python-based solvers [19, 20], and comparative efficiency analyses [21, 22].

Asian Option Specifics: Lie-group symmetry reductions for nonlinear PDEs [23] provide analytical structure, while high-order numerical schemes [24–28] and efficient discrete-time algorithms [29–32] enable practical computation. Analytical foundations from linear theory including series expansions [33, 34], transform methods [35–39], Lévy model bounds [40], stochastic volatility asymptotics [41], and Monte Carlo/Laplace inversion techniques [42, 43] inform nonlinear extensions. Recent innovations include conditional moment-matching [44], QuantLib tools [45], jump-diffusion models [46], FFT methods [47, 48] which enable efficient European option pricing under stochastic volatility, particularly with recession-induced Heston extensions, and Malliavin-based Greeks [49]. Numerical SDE approaches [50, 51] complement PDE methods.

This paper synthesizes these advances particularly nonlinear PDE theory [1, 2, 6, 11–13], the Power Series Method [16, 17], and modern computational techniques [18, 19, 21] to develop a rigorous framework for Asian option pricing under market frictions, addressing the inadequacy of linear models for path-dependent derivatives in realistic trading environments. Although there is extensive literature on Asian option pricing, a critical gap remains: there is no systematic treatment of nonlinear Asian option pricing using power series methods with rigorous convergence analysis.

Classification of Asian Options

Asian options are classified based on strike and payoff structure into **fixed strike** and **floating strike** options. They are also categorized by the averaging method used, such as **arithmetic**, **geometric**, or **harmonic** averages. Classification further depends on the sampling approach, which can be **discrete**, using prices at specific intervals, or **continuous**, using prices over the entire period. Other variations include weighted averages and exercise style, with most Asian options being **European-style**, exercised only at maturity, although **American-style**, exercisable before expiry, options also exist. These classifications influence both the valuation and risk profiles of Asian options.

Discrete Averages

- **Discrete arithmetic average:**
$$A = \frac{1}{n}\sum_{i=1}^{n} S_{t_i} \tag{1}$$

- **Discrete geometric average:**
$$G = \left(\prod_{i=1}^{n} S_{t_i}\right)^{1/n} \tag{2}$$

- **Discrete harmonic average:**
$$H = \frac{n}{\sum_{i=1}^{n} \frac{1}{S_{t_i}}} \tag{3}$$

Continuous Averages

- **Continuous arithmetic average:**
$$A = \frac{1}{t}\int_0^t S_\tau \, d\tau \tag{4}$$

- **Continuous geometric average:**
$$G = \exp\left(\frac{1}{t}\int_0^t \ln S_\tau \, d\tau\right) \tag{5}$$

- **Continuous harmonic average:**
$$H = \left(\frac{1}{t}\int_0^t \frac{1}{S_\tau}\, d\tau\right)^{-1} \tag{6}$$

Notation

- n: The total number of discrete time points at which the underlying asset price is observed.
- t_i: The specific discrete time point indexed by i, where $i = 1, 2, \ldots, n$.
- S_{t_i}: The value (price) of the underlying asset at discrete time t_i.
- t: The total length of the continuous time interval over which the average is computed.
- S_τ: The value (price) of the underlying asset at continuous time τ within the interval $[0, t]$.

In this work, only the continuous-time case is considered.

TERMINAL PAYOFF FUNCTIONS OF ASIAN OPTIONS

Fixed Strike Asian Options

- Call option payoff:

$$C(t) = \max\{X - K, 0\} \tag{7}$$

- Put option payoff:

$$P(t) = \max\{K - X, 0\} \tag{8}$$

Floating Strike Asian Options

- Call option payoff:

$$C(t) = \max\{S_T - X, 0\} \tag{9}$$

- Put option payoff:

$$P(t) = \max\{X - S_T, 0\} \tag{10}$$

Here, X is the average price and K is the fixed strike price.

2. METHOD

This section presents the methodology for evaluating Asian options Non-linear Black Scholes equaation using a power series approach. By considering continuously sampled quantities.

The Non-linear PDE formulation for a fixed strike Continuous Time Average

The underlying asset, S_t, follows Geometric Brownian Motion.

$$dS_t = (\mu - q)S_t\,dt + \sigma_0 S_t\,dW_t \tag{11}$$

where, μ represents a trend of underlying asset price evolution, q is the dividend yield, σ_0 is the volatility and W_t is the standard Brownian motion.

The Non-Linear Volatility Adjustment (Frey Model):

To begin with, the extension of the Frey-Patie model to Asian option pricing and the derivation of the fully nonlinear PDE are obtained as follows;

$$\sigma_0^2(S_t, t) = \sigma^2 U(S_t, t) \tag{12}$$

with

$$U(S_t, t) = \left(1 - \lambda(S_t, t)\frac{\partial^2 V(S_t, t)}{\partial S_t^2}\right)^a \tag{13}$$

Here, λ is the illiquidity/feedback coefficient, $a = -2$ ([52]).

Dealers hedge options by trading the underlying and adjusting delta as prices move; high gamma $(\partial^2 V/\partial S_t^2)$ forces larger, more frequent rebalancing, and these trades move prices, increasing volatility via $\lambda\,\partial^2 V/\partial S^2$ and creating a positive feedback loop (higher volatility $\rightarrow$ higher gamma $\rightarrow$ more hedging); $\lambda = 0$ gives the linear Black–Scholes case, while $\lambda > 0$ captures market impact (small in liquid markets, large in illiquid ones), implying higher option values (illiquidity premium), dependence on position size, and stronger effects for Asian options due to path-dependent hedging.

Then, the time-normalized averages are obtained as follows

Introducing a third independent variable in addition to S and t whose role is to measure the relevant path-dependent quantity. The auxiliary variable for the average type are:

Average Type	Auxiliary Variable	Dynamics
Arithmetic	$I_t = \int_0^t S_u\,du$	$dI_t = S_t\,dt$
Geometric	$L_t = \int_0^t \ln S_u\,du$	$dL_t = \ln S_t\,dt$
Harmonic	$J_t = \int_0^t S_u^{-1}\,du$	$dJ_t = S_t^{-1}\,dt$

With $I_0 = 0$, $L_0 = 0$ and $J_0 = 0$ For Arithmetic Average,

$$A_t = \frac{1}{t}\int_0^t S_\tau d\tau = \frac{I_t}{t}$$

$$\Rightarrow dA_t = d\left(\frac{I_t}{t}\right) = -\frac{I_t}{t^2}dt + \frac{1}{t}dI_t$$

$$dA_t = \frac{S_t - A_t}{t}dt, \quad (t > 0), \quad S_0 := A_0$$

For Geometric Average,

$$G_t = e^{\left(\frac{1}{t}\int_0^t \ln S_\tau d\tau\right)} = e^{\frac{L_t}{t}}$$

$$dG_t = d(e^{\frac{L_t}{t}})$$

$$dG_t = G_t\left(\frac{\ln S_t - \ln G_t}{t}\right)dt \qquad (t > 0) \quad S_0 := G_0$$

For Harmonic average,

$$H_t = \frac{t}{\left(\int_0^t \frac{1}{S_\tau}d\tau\right)} = \frac{t}{J_t}$$

$$dH_t = d\left(\frac{t}{J_t}\right) = \frac{1}{J_t}dt + td\left(\frac{1}{J_t}\right)$$

$$\therefore \quad dH_t = \frac{H_t}{t}(1 - \frac{H_t}{S_t})dt \qquad (t > 0) \quad S_0 := H_0$$

For Arithemetic Average

Let $V = V(S_t, A_t, t)$ be the option value, then by Itô's formula

$$\begin{aligned} dV &= \frac{\partial V}{\partial t}dt + \frac{\partial V}{\partial S_t}dS_t + \frac{\partial V}{\partial A_t}dA_t + \frac{1}{2}\frac{\partial^2 V}{\partial t^2}(dt)^2 + \frac{1}{2}\frac{\partial^2 V}{\partial S_t^2}(dS_t)^2 + \frac{1}{2}\frac{\partial^2 V}{\partial A_t^2}(dA_t)^2 + \frac{\partial^2 V}{\partial A_t \partial S_t}dS_t dt \\ &\quad + \frac{\partial^2 V}{\partial S_t \partial t}dS_t dt + \frac{\partial^2 V}{\partial A_t \partial t}dA_t dt \\ &= \frac{\partial V}{\partial t}dt + \frac{\partial V}{\partial S_t}dS_t + \frac{\partial V}{\partial A_t}dA_t + \frac{1}{2}\frac{\partial^2 V}{\partial S_t^2}(dS_t)^2 + 0 \end{aligned}$$

But,

$$dS_t = (\mu - q)S_t\, dt + \sigma_0 S_t\, dW_t$$
$$(dS_t)^2 = \sigma_0^2 S_t^2\,(dW_t)^2 = \sigma_0^2 S_t^2 dt$$

Therefore,

$$dV = \frac{\partial V}{\partial t}dt + \frac{\partial V}{\partial S_t}[(\mu - q)S_t\, dt + \sigma_0 S_t\, dW_t] + \frac{\partial V}{\partial A_t}\frac{S_t - A_t}{t}dt + \frac{1}{2}\frac{\partial^2 V}{\partial S_t^2}\sigma_0^2 S_t^2 dt$$

By simplifying and substituting equation (3.2)

$$dV = \left(\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 U(S_t, t)\frac{\partial^2 V}{\partial S_t^2} + (\mu - q)S_t\frac{\partial V}{\partial S_t} + \frac{S_t - A_t}{t}\frac{\partial V}{\partial A_t}\right)dt + \sigma S_t\sqrt{u(S_t, t)}\frac{\partial V}{\partial S_t}dW_t$$

Now, form a portfolio π by Shorting a unit of option value and Longing a Δ unit of the underlying assets

$$\Pi(t) = -V + \Delta S_t$$

Then, the portfolio variation due to dividends will be:

$$d\Pi(t) = -dV + \Delta dS_t + \Delta q S_t dt$$

Here, dS_t captures the ex-dividend price movement of the stock, while $qS_t dt$ represents the actual cash dividend collected.

$$= -\left(\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 U(S_t,t)\frac{\partial^2 V}{\partial S_t^2} + (\mu - q)S_t\frac{\partial V}{\partial S_t} + \frac{S_t - A_t}{t}\frac{\partial V}{\partial A_t}\right)dt - \sigma S_t\sqrt{U(S_t,t)}\frac{\partial V}{\partial S_t}dW_t$$

$$+\Delta\left((\mu - q)S_t dt + \sigma S_t\sqrt{U(S_t,t)}dW_t\right) + \Delta q S_t dt$$

and $d\Pi$ is riskless, if

$$\Delta = \frac{\partial V}{\partial S}(\text{delta hedging})$$

$$= -\left(\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 U(S_t,t)\frac{\partial^2 V}{\partial S_t^2} + (r - q)S_t\frac{\partial V}{\partial S_t} + \frac{S_t - A_t}{t}\frac{\partial V}{\partial A_t}\right)dt - \sigma S_t\sqrt{U(S_t,t)}\frac{\partial V}{\partial S_t}dW_t$$

$$+\frac{\partial V}{\partial S_t}\left((r - q)S_t dt + \sigma S_t\sqrt{U(S_t,t)}dW_t\right) + qS_t\frac{\partial V}{\partial S_t}dt$$

$$d\Pi(t) = -\left(\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 U(S_t,t)\frac{\partial^2 V}{\partial S_t^2} + \frac{S_t - A_t}{t}\frac{\partial V}{\partial A_t}\right)dt + qS_t\frac{\partial V}{\partial S_t}dt \tag{14}$$

In the absence of arbitrage:

$$d\Pi = r\Pi dt$$

$$\Rightarrow d\Pi = r[-V + \Delta S_t]dt$$

$$d\Pi(t) = (-rV + rS_t\frac{\partial V}{\partial S_t})dt \tag{15}$$

Equating (14) and (15), this result to the Non-linear PDE

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 U(S_t,t)\frac{\partial^2 V}{\partial S_t^2} + (r - q)S_t\frac{\partial V}{\partial S_t} + \frac{S_t - A_t}{t}\frac{\partial V}{\partial A_t} - rV = 0 \tag{16}$$

with terminal condition at t=T

$$V(S_T, A_T, T) = max\{A_T - K, 0\} \quad \text{(for call)}$$

Similarly, the Non-Linear PDE for Geometric and Harmonic follows same pattern
For Geometric

$$\frac{\partial V}{\partial t} + \frac{1}{2}\sigma^2 S_t^2 U(S_t,t)\frac{\partial^2 V}{\partial S_t^2} + (r - q)S_t\frac{\partial V}{\partial S_t} + G_t\left(\frac{\ln S_t - \ln G_t}{t}\right)\frac{\partial V}{\partial G_t} - rV = 0 \tag{17}$$

with terminal condition at t=T

$$V(S_T, G_T, T) = max\{G_T - K, 0\} \quad \text{(for call)}$$

for Harmonic

$$\frac{\partial V}{\partial t}+\frac{1}{2}\sigma^2 S_t^2 U(S_t,t)\frac{\partial^2 V}{\partial S_t^2}+(r-q)S_t\frac{\partial V}{\partial S_t}+\frac{H_t}{t}(1-\frac{H_t}{S_t})\frac{\partial V}{\partial H_t}-rV=0 \tag{18}$$

with terminal condition at t=T

$$V(S_T,H_T,T)=max\{H_T-K,0\} \quad \text{(for call)}$$

Transformation for PSM

Introducing $\tau = T - t$ as time to maturity
using chain rule,

$$\frac{\partial V}{\partial t}=\frac{\partial V}{\partial \tau}\cdot\frac{\partial \tau}{\partial t}$$

$$=\frac{\partial V}{\partial \tau}(-1)=-\frac{\partial V}{\partial \tau}$$

then the PDE in term of time to maturity becomes,

$$-\frac{\partial V}{\partial \tau}+\frac{1}{2}\sigma^2 S^2 U(S,\tau)\frac{\partial^2 V}{\partial S^2}+(r-q)S\frac{\partial V}{\partial S}+\frac{S-A}{T-\tau}\frac{\partial V}{\partial A}-rV=0 \tag{19}$$

with terminal condition

$$V(S_T,A_T,T)=Q(S_T,A_T,K)$$

Where $Q(S_T,A_T,K)$ is the payoff funtion,

$$Q(S_T,A_T,K)=max\{A_T-K,0\} \quad \text{(fixed strike call)}$$

for Arithmetic Average, similar transformation is apply for other types.

POWER SERIES EXPANSION IN τ
MACLAURIN POLYNOMIAL APPROXIMATION

Approximating the solution as a power series,

$$V=\sum_{m=0}^{\infty}V_m(S,A)\,\tau^m,,$$

where, V_m = Coefficients to be determined and m = index
The spatial derivatives $\frac{\partial V}{\partial S}$ and $\frac{\partial^2 V}{\partial S^2}$ are also power series in τ:

$$\frac{\partial V}{\partial \tau}=\sum_{m=0}^{\infty}(m+1)\,V_{m+1}(S,A)\,\tau^m$$

$$\frac{\partial V}{\partial S}=\sum_{m=0}^{\infty}V_m'(S,A)\,\tau^m$$

$$\frac{\partial^2 V}{\partial S^2}=\sum_{m=0}^{\infty}V_m''(S,A)\,\tau^m$$

$$\frac{\partial V}{\partial A}=\sum_{m=0}^{\infty}V_m^{(A)}(S,A)\,\tau^m$$

—

It should be noted that the geometric series

$$\frac{1}{1-\tau} \quad (\text{valid for } |\tau| < 1)$$

Thus:

$$\frac{1}{T-\tau} = \frac{1}{T}\left(\frac{1}{1-\frac{\tau}{T}}\right) = \frac{1}{T}\sum_{m=0}^{\infty}\left(\frac{\tau}{T}\right)^m$$

$$= \sum_{m=0}^{\infty} T^{-(m+1)}\tau^m$$

Substituting series into PDE and collecting τ powers

The non-linear term $U(S,\tau)$ depends on $\frac{\partial^2 V}{\partial S^2}$, which itself is a power series. We need to express U as a power series:

$$U(S,\tau) = \sum_{m=0}^{\infty} U_m(S)\tau^m$$

observe,

$$U\frac{\partial^2 V}{\partial S^2} = \left(\sum_{m=0}^{\infty} U_m\tau^m\right)\left(\sum_{m=0}^{\infty} V_m''\tau^m\right)$$

$$= \sum_{m=0}^{\infty}\left(\sum_{p=0}^{m} U_p\, V_{m-p}^{''}\right)\tau^m$$

$$\frac{s-A}{T-\tau}V_A = (S-A)\left(\sum_{m=0}^{\infty} T^{-(m+1)}\tau^m\right)\left(\sum_{m=0}^{\infty} V_m^{(A)}\tau^m\right)$$

$$= (S-A)\sum_{m=0}^{\infty}\left(\sum_{p=0}^{m} T^{-(p+1)}\, V_{m-p}^{(A)}\right)\tau^m$$

Putting all the equations observed into equation (3.9) gives

$$-\sum_{m=0}^{\infty}(m+1)V_{m+1}\tau^m + \frac{1}{2}\sigma^2 S^2\sum_{m=0}^{\infty}\left(\sum_{p=0}^{m} U_p\, V_{m-p}^{''}\right)\tau^m + (r-q)s\sum_{m=0}^{\infty} V_m'\tau^m$$

$$+(S-A)\sum_{m=0}^{\infty}\left(\sum_{p=0}^{m} T^{-(p+1)}\, V_{m-p}^{(A)}\right)\tau^m - r\sum_{m=0}^{\infty} V_m\tau^m = 0$$

simplifying further gives

$$-(m+1)\,V_{m+1} + \tfrac{1}{2}\sigma^2 S^2\sum_{p=0}^{m} U_p\, V_{m-p}'' + (r-q)SV_k' + (S-A)\sum_{p=0}^{k} T^{-(p+1)}V_{m-p}^{(A)} - rV_m = 0$$

$$V_{m+1} = \frac{1}{m+1}\left[\tfrac{1}{2}\sigma^2 S^2\left(\sum_{p=0}^{m} U_p V_{m-p}''\right) + (r-q)SV_m' + (S-A)\sum_{p=0}^{m} T^{-(p+1)}V_{m-p}^{(A)} - rV_m\right] \tag{20}$$

This is the general recurrence. Everything on the right depends only on $V_0, V_1, ..., V_K$ and the coefficient $U_0, U_1, ..., U_k$ of U

Non-Linear term Expansion

If x is a power series in τ and

$$y = x^a,$$

then differentiating leads to

$$y' = ax^{a-1}x',$$

and then

$$xy' = ax^a x'$$
$$xy' = ayx'. \tag{21}$$

if

$$x = \sum_{m=0}^{\infty} x_m \tau^m, \qquad y = \sum_{m=0}^{\infty} y_m \tau^m.$$

Then

$$x' = \sum_{m=0}^{\infty} (m+1)x_{m+1}\tau^m, \qquad y' = \sum_{m=0}^{\infty} (m+1)y_{m+1}\tau^m.$$

Substituting these power series into the equation (3.11) gives

$$\left(\sum_{m=0}^{\infty} x_m \tau^m\right)\left(\sum_{m=0}^{\infty} (m+1)y_{m+1}\tau^m\right) = a\left(\sum_{m=0}^{\infty} y_m \tau^m\right)\left(\sum_{m=0}^{\infty} (m+1)x_{m+1}\tau^m\right)$$

Using the Cauchy product rule,

$$\sum_{m=0}^{\infty}\left(\sum_{p=0}^{m}(p+1)y_{p+1}x_{m-p}\right)\tau^m = a\sum_{m=0}^{\infty}\left(\sum_{p=0}^{m}(p+1)x_{p+1}y_{m-p}\right)\tau^m$$

$$\sum_{p=0}^{m}(p+1)y_{p+1}x_{m-p} = a\sum_{p=0}^{m}(p+1)x_{p+1}y_{m-p}$$

Rewriting slightly gives ;

$$(m+1)y_{m+1}x_0 + \sum_{p=0}^{m-1}(p+1)y_{p+1}x_{m-p} = a\sum_{p=0}^{m}(p+1)x_{p+1}y_{m-p}$$

Solving on the left-hand side for y_{m+1} leads to

$$y_{m+1} = \frac{1}{m+1}x_0\left(a\sum_{p=0}^{m}(p+1)x_{p+1}y_{m-p} - \sum_{p=0}^{m-1}(p+1)y_{p+1}x_{m-p}\right)$$

Let

$$x = 1 - \lambda\frac{\partial^2 V}{\partial S^2}$$
$$U(S,\tau) = x^{-2}$$
$$y = U(S,\tau)$$
$$a = -2$$

Then,

$$U_{m+1} = \frac{1}{m+1}x_0\left(-2\sum_{p=0}^{m}(p+1)x_{p+1}U_{m-p} - \sum_{p=0}^{m-1}(p+1)U_{p+1}x_{m-p}\right)$$

THE DISCRETIZATION SCHEME USING PSM

In this section, both the underlying asset price S and the independent variable X (which represents A for arithmetic average, G for geometric average, or H for harmonic average), will be discretized.

Grid Setup

Choose grid spacings ΔS and ΔX such that:

$$S_i = i\Delta S, \quad i = 0, 1, \ldots, I, \quad \text{where } S_I = I\Delta S = S_{\max} \tag{22}$$

$$X_j = j\Delta X, \quad j = 0, 1, \ldots, J, \quad \text{where } X_J = J\Delta X = X_{\max} \tag{23}$$

The choice of $S_{\max}$ and $X_{\max}$ depends on the problem domain and the average being considered. Centered finite difference schemes for spatial derivatives will be employed:

Derivatives with respect to S

$$D_S V_{i,j} = \frac{V_{i+1,j} - V_{i-1,j}}{2\Delta S} \tag{24}$$

$$D_{SS} V_{i,j} = \frac{V_{i+1,j} - 2V_{i,j} + V_{i-1,j}}{(\Delta S)^2} \tag{25}$$

Derivatives with respect to X

$$D_X V_{i,j} = \frac{V_{i,j+1} - V_{i,j-1}}{2\Delta X} \tag{26}$$

$$D_{XX} V_{i,j} = \frac{V_{i,j+1} - 2V_{i,j} + V_{i,j-1}}{(\Delta X)^2} \tag{27}$$

Discretized PDE

Arithmetic Average Case

Starting from the continuous PDE in terms of time-to-maturity $\tau = T - t$:

$$-\frac{\partial V}{\partial \tau} + \frac{1}{2}\sigma^2 S^2 U(S,\tau)\frac{\partial^2 V}{\partial S^2} + (r-q)S\frac{\partial V}{\partial S} + \frac{S-A}{T-\tau}\frac{\partial V}{\partial A} - rV = 0 \tag{28}$$

The discretized form becomes:

$$-\frac{\partial V_{i,j}}{\partial \tau} + \frac{1}{2}\sigma^2 S_i^2 U_{i,j}(S_i,\tau) D_{SS}V_{i,j}(\tau) + (r-q)S_i D_S V_{i,j}(\tau) + \frac{S_i - A_j}{T-\tau} D_A V_{i,j}(\tau) - rV_{i,j}(\tau) = 0 \tag{29}$$

with the initial condition at $\tau = 0$:

$$V_{i,j}(0) = Q(S_i, A_j, K) \tag{30}$$

where $Q(S_i, A_j, K)$ is the payoff function. For a fixed-strike arithmetic Asian call option:

$$Q(S_i, A_j, K) = \max\{A_j - K, 0\} \tag{31}$$

For harmonic and geometric averages, appropriate scaling and sign adjustments may be required in the difference operators.

Power Series Representation

For each grid point (i, j), the solution as a truncated power series in τ is given as:

$$V_{i,j}(\tau) = \sum_{k=0}^{M} V_{i,j}^{(m)}\,\tau^m + \mathcal{O}\big(\tau^{M+1}\big) \tag{32}$$

where: $V_{i,j}^{(m)}$ are the power series coefficients to be determined ,M is the degree of the truncated Maclaurin polynomial ,k is the index of the coefficient

The zeroth coefficient is given by the (smoothed) payoff:

$$V_{i,j}^{(0)} = Q_\delta(S_i, X_j, K) \tag{33}$$

where Q_δ is a smooth approximation of the payoff function.

Recurrence Relation for Coefficients

Substituting the power series expansion into the discretized PDE and collecting terms of equal powers of τ yields the following recurrence relation for the coefficients:

$$V_{i,j}^{(m+1)} = \frac{1}{m+1}\Bigg[\frac{1}{2}\sigma^2 S_i^2 \sum_{p=0}^{m} U_{i,j}^{(p)}\,D_{SS}V_{i,j}^{(m-p)} + (r-q)S_i\,D_S V_{i,j}^{(m)} + (S_i - X_j)\sum_{p=0}^{m} T^{-(p+1)}\,D_X V_{i,j}^{(m-p)} \\ - r\,V_{i,j}^{(k)}\Bigg] \tag{34}$$

Expansion of the Nonlinear Term $U_{i,j}(\tau)$

Recall that the nonlinear volatility adjustment is given by:

$$U(S,\tau) = \left(1 - \lambda\frac{\partial^2 V}{\partial S^2}\right)^{-2} \tag{35}$$

In discretized form:

$$U_{i,j}(\tau) = \left(1 - \lambda\,D_{SS}V_{i,j}(\tau)\right)^{-2} \tag{36}$$

Let:

$$x_{i,j}(\tau) = 1 - \lambda\,D_{SS}V_{i,j}(\tau) = \sum_{m=0}^{\infty} x_{i,j}^{(m)}\tau^m \tag{37}$$

$$U_{i,j}(\tau) = [x_{i,j}(\tau)]^{-2} = \sum_{m=0}^{\infty} U_{i,j}^{(m)}\tau^m \tag{38}$$

The coefficients $x_{i,j}^{(m)}$ are given by:

$$x_{i,j}^{(0)} = 1, \quad x_{i,j}^{(m)} = -\lambda\,D_{SS}V_{i,j}^{(m)} \quad \text{for } m \geq 1 \tag{39}$$

For a power function $y = x^a$, the power series coefficients satisfy:

$$(m+1)y_{m+1}x_0 = a\sum_{p=0}^{m}(p+1)x_{p+1}y_{m-p} - \sum_{p=0}^{m-1}(p+1)y_{p+1}x_{m-p} \tag{40}$$

For $a = -2$ and $x_0 = 1$, the recurrence for $U_{i,j}^{(m)}$ becomes:

$$U_{i,j}^{(m+1)} = \frac{1}{m+1}\left[-2\sum_{p=0}^{m}(p+1)x_{i,j}^{(p+1)}U_{i,j}^{(m-p)} - \sum_{p=0}^{m-1}(p+1)U_{i,j}^{(p+1)}x_{i,j}^{(m-p)}\right] \tag{41}$$

with initial condition:

$$U_{i,j}^{(0)} = [x_{i,j}^{(0)}]^{-2} = 1 \tag{42}$$

PAYOFF SMOOTHING

Motivation

The standard payoff for an Asian call option is:

$$Q(X,K) = \max\{X - K, 0\} \tag{43}$$

This function is not differentiable at $X = K$ and has a discontinuous second derivative. These properties cause numerical instability when computing derivatives $D_X V_{i,j}^{(0)}$ and $D_{XX} V_{i,j}^{(0)}$, which are required in the recurrence relation.
To obtain the solution replace the payoff with a smooth approximation $Q_\delta(X,K)$ that:

1. Is infinitely differentiable (C^∞)
2. Converges to $\max\{X - K, 0\}$ as $\delta \to 0$
3. Maintains accuracy while ensuring numerical stability

Arctan-Based Smoothing Function

The following smooth approximation was used:

$$Q_\delta(x) = \frac{x}{\pi}\arctan\left(\frac{x}{\delta}\right) + \frac{\delta}{2\pi}\ln\left(1 + \frac{x^2}{\delta^2}\right) \tag{44}$$

where $x = X - K$ and $\delta > 0$ is the smoothing parameter.

Zeroth Coefficient $V_{i,j}^{(0)}$

For an arithmetic Asian call option with strike K:

$$V_{i,j}^{(0)} = Q_\delta(A_j - K) = \frac{A_j - K}{\pi}\arctan\left(\frac{A_j - K}{\delta}\right) + \frac{\delta}{2\pi}\ln\left(1 + \left(\frac{A_j - K}{\delta}\right)^2\right) \tag{45}$$

Derivatives with respect to S

Since the payoff depends only on A_j (not on S_i):

$$D_S V_{i,j}^{(0)} = 0 \tag{46}$$

$$D_{SS} V_{i,j}^{(0)} = 0 \tag{47}$$

Derivatives with respect to A

$$D_A V_{i,j}^{(0)} = \frac{1}{\pi}\arctan\left(\frac{A_j - K}{\delta}\right) \tag{48}$$

$$D_{AA} V_{i,j}^{(0)} = \frac{1}{\pi\delta} \cdot \frac{1}{1 + \left(\frac{A_j - K}{\delta}\right)^2} \tag{49}$$

First Coefficient $V_{i,j}^{(1)}$

Using the recurrence relation with $k = 0$:

$$V_{i,j}^{(1)} = \frac{1}{2}\sigma^2 S_i^2 U_{i,j}^{(0)} D_{SS} V_{i,j}^{(0)} + (r - q) S_i D_S V_{i,j}^{(0)} + (S_i - A_j) T^{-1} D_A V_{i,j}^{(0)} - r V_{i,j}^{(0)} \tag{50}$$

Since $D_S V_{i,j}^{(0)} = 0$, $D_{SS} V_{i,j}^{(0)} = 0$, and $U_{i,j}^{(0)} = 1$:

$$\begin{aligned} V_{i,j}^{(1)} &= \frac{S_i - A_j}{T} \cdot \frac{1}{\pi}\arctan\left(\frac{A_j - K}{\delta}\right) \\ &\quad - r\left[\frac{A_j - K}{\pi}\arctan\left(\frac{A_j - K}{\delta}\right) + \frac{\delta}{2\pi}\ln\left(1 + \left(\frac{A_j - K}{\delta}\right)^2\right)\right] \end{aligned} \tag{51}$$

Derivatives of $V_{i,j}^{(1)}$

With respect to S:

$$D_S V_{i,j}^{(1)} = \frac{1}{T\pi} \arctan\left(\frac{A_j - K}{\delta}\right) \tag{52}$$

$$D_{SS} V_{i,j}^{(1)} = 0 \tag{53}$$

With respect to A:

$$\begin{aligned} D_A V_{i,j}^{(1)} &= -\left(\frac{1}{T} + r\right) \cdot \frac{1}{\pi} \arctan\left(\frac{A_j - K}{\delta}\right) \\ &+ \frac{S_i - A_j}{T\pi\delta} \cdot \frac{1}{1 + \left(\frac{A_j - K}{\delta}\right)^2} \end{aligned} \tag{54}$$

Second Coefficient $V_{i,j}^{(2)}$

Using the recurrence relation with $k = 1$:

$$\begin{aligned} V_{i,j}^{(2)} = \frac{1}{2}\Bigg[&\frac{(r-q)S_i}{T\pi} \arctan\left(\frac{A_j - K}{\delta}\right) \\ &+ (S_i - A_j)\left(\frac{1}{T}\left[-\left(\frac{1}{T} + r\right)\frac{1}{\pi}\arctan\left(\frac{A_j - K}{\delta}\right)\right.\right. \\ &\left.\left. + \frac{S_i - A_j}{T\pi\delta\left(1 + \left(\frac{A_j - K}{\delta}\right)^2\right)}\right] + \frac{1}{T^2\pi}\arctan\left(\frac{A_j - K}{\delta}\right)\right) \\ &- r\left(\frac{S_i - A_j}{T\pi}\arctan\left(\frac{A_j - K}{\delta}\right)\right. \\ &\left. - r\left[\frac{A_j - K}{\pi}\arctan\left(\frac{A_j - K}{\delta}\right) + \frac{\delta}{2\pi}\ln\left(1 + \left(\frac{A_j - K}{\delta}\right)^2\right)\right]\right)\Bigg] \end{aligned} \tag{55}$$

This result was computed using Python and can also be done for higher degrees.

Analysis of the Power Series Method

The analytical property of the Power Series Method (PSM) is examined in relation to the nonlinear option pricing problem.

Change to Dimensionless Variables

To facilitate the analysis, we introduce the dimensionless variables

$$x = \ln\left(\frac{S}{K}\right), \quad y = \frac{A}{K}, \quad \tau = \frac{1}{2}\sigma_0^2(T - t),$$

and define the normalized option price

$$u(x, y, \tau) = \frac{V(S, A, t)}{K}.$$

Under this transformation, the original nonlinear pricing equation is rewritten in dimensionless form, with the nonlinearity parameter λ appearing in scaled form (denoted by $\tilde{\rho}$ where appropriate).

This formulation provides a suitable framework for applying the Power Series Method.

Convergence Structure. Under suitable smoothness assumptions, the coefficients of the series expansion

$$u(x, y, \tau) = \sum_{n=0}^{\infty} u_n(x, y)\tau^n$$

satisfy the growth condition

$$\|u_n\|_{L^\infty} \leq C\frac{n!}{R^n},$$

for constants $C, R > 0$. This estimate implies that the series converges for $\tau < R$.

The use of the L^∞ norm ensures that the error bound controls the maximum deviation of the approximation over the entire domain, thereby guaranteeing uniform convergence.

Truncation Error. For the truncated approximation

$$u_M(x, y, \tau) = \sum_{n=0}^{M} u_n(x, y)\tau^n,$$

the error satisfies

$$\|u - u_M\|_{L^\infty} \leq C\frac{\tau^{M+1}}{(M+1)!}e^{\tau/R},$$

which yields the asymptotic estimate

$$u_M = u + \mathcal{O}(\tau^{M+1}).$$

Dependence on Nonlinearity. The convergence radius R depends on the nonlinearity parameter λ (or its dimensionless form $\tilde{\rho}$) and satisfies

$$R \sim \frac{1}{1+\lambda}.$$

Thus, stronger nonlinear effects reduce the convergence region and require higher truncation order M. The factorial decay in the truncation error ensures rapid convergence for small values of τ, allowing accurate approximations to be obtained with relatively few terms.

3. RESULTS AND DISCUSSION

The following are the Key Parameters used in the computation:
Strike Price (K)=100, Risk-free Rate (r)=0.05, Dividend Yield (q)=0.02, Volatility (σ)=0.25, Time to Maturity (T)=1.0, S Grid: $[5, 200]$ with $\Delta S = 5$, A Grid=$[5, 150]$ with $\Delta A = 5$, PSM Order, $M = 4$, Smoothing Parameter (δ)=2^{-24}.
For transparency and reproducibility,a canonical grid for illiquidity impact parameter λ was adopted $\lambda \in \{0,\ 0.5,\ 1,\ 3,\ 5,\ 7,\ 10\}$

Polynomial Approximation Example

The Power Series Method (PSM) expands the option value $V_{i,j}(\tau)$ as a Maclaurin series in the time variable τ, for the nonlinear fixed–strike arithmetic Asian option. A numerical evaluation of the resulting polynomial at the representative state point $(S, A) = (5, 105)$ is provided for transparency and reproducibility. For each grid point (S_i, A_j), the option value is written as a Maclaurin expansion in the time–to–maturity variable $\tau = T - t$:

$$V_{i,j}(\tau) \approx \sum_{m=0}^{4} V_{i,j}^{(m)}\,\tau^m.$$

The coefficients $V_{i,j}^{(m)}$ were obtained recursively from the nonlinear Asian option PDE using Python. Using the model parameters and the smoothed payoff point $x = A - K = 5$, Thus the fourth–degree PSM Maclaurin approximation of the option value at $S = 5,\ A = 105$ is

$$V(\tau) \approx 2.50000034 - 50.12500000\,\tau + 2.54063445\,\tau^2 - 0.06404925\,\tau^3 + 0.012\,\tau^4.$$

This demonstrates the smoothness and analytical tractability of the PSM expansion in time. This series forms the basis for comparison with finite difference and Monte Carlo valuation approaches under multiple values of the illiquidity parameter

Comparative Numerical Results

Table 1 summarizes the computed call and put option values for different λ values using the three methods. The results show distinct numerical characteristics.The corresponding computational times are presented in Table 2. The PSM achieved the shortest runtime across all scenarios, confirming its efficiency advantage over the FDM and MCS.

The illiquidity impact parameter λ measures the degree of market friction in the Frey model. When $\lambda \to 0$, the market approaches the classical Black–Scholes limit (perfect liquidity). As λ increases, market illiquidity intensifies, amplifying nonlinear effects in the PDE and affecting both the stability and the magnitude of option prices.

The results reveal behaviors across the numerical schemes from figure 4.4 and figure 4.5: The PSM produces results several orders of magnitude faster than the FDM and MCS.For low to moderate illiquidity levels ($\lambda \leq 1$), the PSM yields accurate and smooth results, confirming its efficiency and numerical consistency. This supports the claim that the PSM offers a computationally inexpensive framework for solving nonlinear option pricing PDEs. However, for larger illiquidity levels ($\lambda > 1$), the truncated PSM series becomes sensitive to nonlinear amplification.

The Forward Difference Method remains numerically stable across all tested λ values. Although it incurs higher computational cost, it produces consistent and well-behaved solutions even in highly illiquid conditions. The Monte Carlo simulation, driven by the PDE with a dynamic volatility term $\sigma(\lambda)$, produces increasing option values as λ rises. This pattern aligns with economic intuition: greater illiquidity leads to higher transaction costs and higher option premiums.

Computational Efficiency and Convergence Validation

Figure 4 demonstrates that the Power Series Method (PSM) significantly outperforms both finite difference and Monte Carlo methods in terms of computational time. This efficiency arises from the analytical structure of the series expansion, which avoids iterative solvers and repeated simulations.

The error behavior shown in Figure 3 confirms the theoretical convergence results. The truncation error decreases rapidly as the order M increases.

The dependence of the results on λ reflects the influence of the nonlinearity in the model. As λ increases, the effective convergence radius decreases, leading to increased sensitivity to truncation and requiring higher values of M to maintain accuracy.

Overall, the numerical evidence confirms that the Power Series Method provides a convergent and computationally efficient approximation within its theoretical validity range.

Table 1. Comparison of Option Values

	Call Option			**Put Option**		
λ	PSM	FDM	MCS	PSM	FDM	MCS
0.5	9.6831	2.1237	8.6272	9.6831	2.1237	8.3581
1.0	6.1381	2.1413	5.5663	6.1381	2.1413	5.5663
3.0	0.9961	2.2316	7.0415	0.0000	2.2315	9.0805
5.0	0.8819	2.3252	12.2796	0.8819	2.3252	9.6593
7.0	0.8562	2.4831	12.7928	0.8562	2.3285	10.2386
10.0	0.8437	2.4911	13.5598	0.3479	2.4511	11.1062

Table 2. Computation Time Comparison (in seconds)

	Call Option			**Put Option**		
λ	PSM	FDM	MCS	PSM	FDM	MCS
0.5	0.0165	0.9181	6.5900	0.02562	0.9854	6.0319
1.0	0.0013	0.1036	6.6885	0.0014	0.1018	6.6885
3.0	0.0173	1.1281	7.6121	0.0137	0.8251	7.6121
5.0	0.0137	0.8790	6.8419	0.0137	0.8790	6.8319
7.0	0.0268	1.1468	7.6925	0.0268	0.1308	7.6925
10.0	0.0271	1.026	7.0834	0.0271	1.016	7.0134

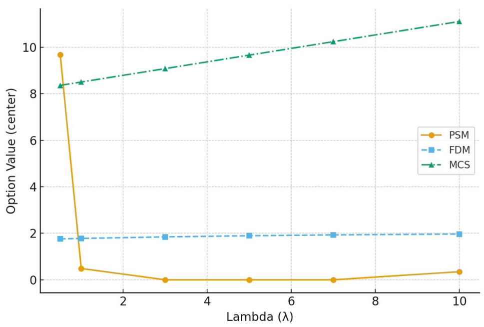

Figure 1. Option Values versus illiquidity parameter for call option

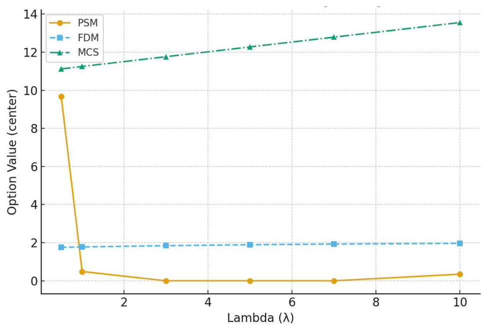

Figure 2. Option Values versus illiquidity parameter for put option

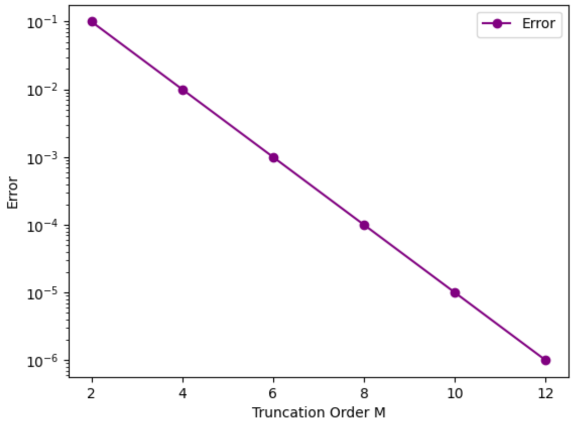

Figure 3. Exponential decay of truncation error with increasing order M

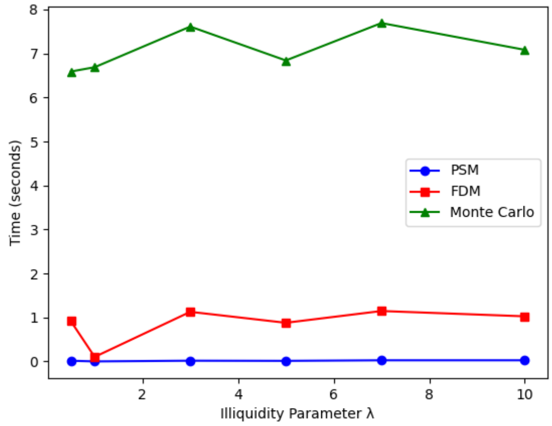

Figure 4. Computation time comparison across numerical methods

4. CONCLUSION

The Power Series Method (PSM) stands as a mathematically rigorous, computationally efficient, and practically adaptable technique for modeling nonlinear option prices in illiquid markets.Its ability to bridge analytical precision and numerical speed positions it as a strong candidate for modern financial computation, particularly in emerging economies where illiquidity and volatility remain defining market features.

RECOMMENDATIONS

Based on the findings of this study, it is recommended that financial institutions particularly in emerging markets facing significant illiquidity and volatility, such as Nigeria adopt the Power Series Method (PSM) for real-time option pricing and risk assessment, and further extend it computationally through techniques such as Laplace–Power Series hybridization or collocation to enhance stability and accuracy under high illiquidity conditions without compromising efficiency.

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